

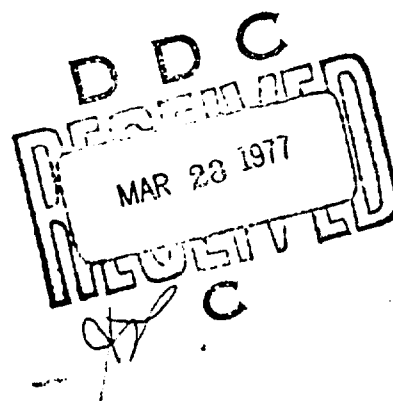
APL/JHU  
TG 1305  
DECEMBER 1976  
Copy No. 1

ADA 037398

*Technical Memorandum*

**THE TRANSIT SYSTEM, 1975**

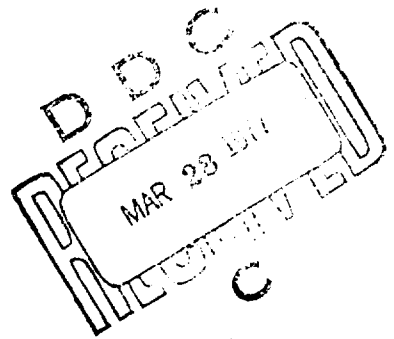
H. D. BLACK  
R. E. JENKINS  
L. L. PRYOR



THE JOHNS HOPKINS UNIVERSITY ■ APPLIED PHYSICS LABORATORY

DDC FILE COPY

## REPORT DOCUMENTATION PAGE

1. REPORT NUMBER APL/JHU/TG-1305 ✓	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) THE TRANSIT SYSTEM, 1975 .	5. TYPE OF REPORT & PERIOD COVERED Technical Memorandum	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) H. D./Black, R. E./Jenkins, and L. L./Pryor	8. CONTRACT OR GRANT NUMBER(s) N00017-72-C-4401 ✓	
9. PERFORMING ORGANIZATION NAME & ADDRESS The Johns Hopkins University Applied Physics Laboratory ✓ Johns Hopkins Rd. Laurel, MD 20810	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Task S1	
11. CONTROLLING OFFICE NAME & ADDRESS Strategic Systems Project Office SP-24 Crystal Mall 3, Arlington, VA 20376	12. REPORT DATE December 1976	13. NUMBER OF PAGES 63
14. MONITORING AGENCY NAME & ADDRESS Naval Plant Representative Office Johns Hopkins Rd. Laurel, MD 20810	15. SECURITY CLASS. (of this report) Unclassified	15. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution is unlimited.	na	
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report) na		
18. SUPPLEMENTARY NOTES na		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Doppler Navigation Satellite navigation Transit system		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This report describes the development and 1975 status of the Transit System (Navy Navigation Satellite System). Transit has been available for military use since 1963 and for public use since 1967, providing all-weather navigation to world-wide users. For users on land, the system provides a standard for global surveying data. A single 15-min data span provides the necessary data for an on-land user to obtain his position with a 20- to 40-m precision. If all the data available in a 2-day period are used, then the position is good to a 1- to 5-m precision. The position is specified in a global datum system and is free of the limitations associated with local datum coordinates. At the time of this writing, five satellites were in polar orbits. The system has been continuously updated and improved with no user changes required. In late 1975, the geopotential model used for the previous 7 years was replaced by the DoD model WGS-72. The importance of the change to system users is discussed.		

APL/JHU

TG 1305

DECEMBER 1976

*Technical Memorandum*

**THE TRANSIT SYSTEM, 1975**

H. D. BLACK

R. E. JENKINS

L. L. PRYOR

THE JOHNS HOPKINS UNIVERSITY ■ APPLIED PHYSICS LABORATORY

Johns Hopkins Road, Laurel, Maryland 20810

Operating under Contract N00017-72-C-4401 with the Department of the Navy



# CONTENTS

List of Illustrations . . . . .	6
List of Tables . . . . .	7
1. Introduction . . . . .	9
2. How the System Works . . . . .	11
3. Current Status . . . . .	14
4. Opportunities for Obtaining a Fix . . . . .	15
5. Accuracy and Quality Control . . . . .	16
6. Broadcast Ephemerides . . . . .	18
7. Satellite Clock . . . . .	20
8. Geopotential Model . . . . .	22
9. At-sea Navigation . . . . .	24
10. Planned Improvements . . . . .	28
10.1 Orbit-determination Software Changes . . . . .	28
10.2 Hardware Changes . . . . .	29
11. WGS-72 Geopotential Model . . . . .	30
11.1 Comparison with Other Geopotential Models . . . . .	32
12. Geoid Height Contour Map . . . . .	37
13. Sun- and Moon-induced Earth Body-tides . . . . .	42
14. Radiation Pressure . . . . .	46
15. Drag Compensation (DISCOS) . . . . .	48
16. Flight Computer and Extra Message . . . . .	50
17. Clock Improvements . . . . .	51
Acknowledgment . . . . .	54
References . . . . .	55
Bibliography . . . . .	59
Appendix . . . . .	61

ACCESSION for	
NTIS	White Section <input checked="" type="checkbox"/>
DDC	Buff Section <input type="checkbox"/>
UNANNOUNCED	<input type="checkbox"/>
JUSTIFICATION	
BY	
DISTRIBUTION/AVAILABILITY CODES	
Dist.	AVAIL. ST. or SPECIAL

# ILLUSTRATIONS

1	Schematic of the Transit System . . . . .	12
2	Transit Surveying Results . . . . .	17
3	Satellite 1967-92a Clock Error During 1973 . . . . .	20
4	Satellite Position Error Caused by Truncating the Geopotential Coefficients at Degree n, Satellite at 1100-km Altitude . . . . .	23
5	Orbit Error for WGS-72 and GEM-1 Geopotential Models . . . . .	35
6	Orbit Error for WGS-72 and GEM-6 Geopotential Models . . . . .	35
7	Orbit Error for WGS-72 and APL 4.5 Geopotential Models . . . . .	36
8	Orbit Error for WGS-72 and SE 3 Geodesy . . . . .	36
9	Transit Geoid Map . . . . .	38
10	Antenna Height Compensation . . . . .	39
11	Determination of the 3-Dimensional Position (all satellites, days 40-43, 1974) . . . . .	41
12	Expected Along-track Residuals (lunar body-tide effect) . . . . .	44
13	Orbit Determination with and without Lunar Body-tide Effect . . . . .	45
14	Comparison Between Observed and Simulated Body-tide Error . . . . .	45
15	TIP Satellite, Artist's Concept . . . . .	49
16	IPS Clock Control System . . . . .	52

TABLES

1	Satellites in Service, May 1975 . . . . .	14
2	Time and Frequency Errors at the Transit Tracking Sites . . . . .	21
3	Precision of Satellite Orbit . . . . .	22
4	Data on Roll and Pitch Periods of Ships . . . . .	26
5	Changes to the Transit Stations' Coordinates . . . . .	29
6	Transit System - Surveyor's Single-Pass Error Budget . . . . .	30
7	Satellite 1967-48a . . . . .	33

## 1. INTRODUCTION

The Transit System (the Navy Navigation Satellite System) was invented by F. T. McClure (Ref. 1). McClure had before him the new discovery of satellite orbit determination using the Doppler shift measurement (Ref. 2). The invention of Transit came from the realization that the solution of the orbit problem, using Doppler shift measurements, could be inverted: If the orbit is known and the measurement-site location is unknown, then the location could be found from measurements of the Doppler shift. That the solution is unique cannot be seen in any simple, analytic way (Ref. 3). The development of the system proceeded at The Johns Hopkins University Applied Physics Laboratory (APL) under DoD support and first met its operational specification (to a precision of 0.1 nmi navigation) in 1963.

Transit differs strikingly from older forms of navigation:

1. Unlike celestial navigation, no skill or special knowledge is required of the navigator; he simply reads the automatically produced result.
2. Directional antennas are not required. A simple whip suffices.
3. It was the first navigation/surveying system to employ an internally consistent global datum.
4. Most important, the system is immune to the vagaries of the weather, nor does its success depend upon whether it is night or day.

Although an explanation of how the system works can be couched in a geometrical (intersecting hyperboloids) framework (Ref. 4, especially p. 140), the "line-of-position" graphic technique so familiar to celestial navigators is not useful in solving

---

Ref. 1. F. T. McClure, Method of Navigation, U.S. Patent No. 3,172,108, filed 12 May 1958, issued 2 March 1965.

Ref. 2. W. H. Guier and G. C. Weiffenbach, "Theoretical Analysis of Doppler Radio Signals from Earth Satellites," APL/JHU BB-276, 1958.

Ref. 3. W. H. Guier and G. C. Weiffenbach, "A satellite Doppler Navigation System," Proc. IRE, Vol. 48, pp. 407-516, 1960.

Ref. 4. R. B. Kershner and R. R. Newton, "The TRANSIT System," J. Inst. Nav., Vol. 15, pp. 129-144 (see p. 140), 1962.



the associated set of simultaneous equations. One reason for this is that the fix computation (necessarily) produces a frequency calibration (of the navigator's standard), as well as the latitude and longitude.

Transit has been used continuously since 1963 and has been continuously improved. The system was released for public use by Presidential directive in 1967.

If the reader is familiar with the Transit System and only interested in the newer aspects, he may turn to Section 10.

## 2. HOW THE SYSTEM WORKS

Figure 1 shows schematically the overall "structure" of the system.

1. There are a number of satellites (5 at the present writing) in near-earth orbits. All orbits are (approximately) polar and circular with an altitude of approximately 1100 km. Each satellite contains
  - a. A highly precise frequency standard that "drives" two transmitters nominally at 150 MHz and 400 MHz. A counter driven by this same standard functions as a satellite clock.
  - b. A core memory containing a current ephemeris of the satellite. (The ephemeris information and the clock control register can be revised from the ground via a command link. The satellite is stabilized so that the antennas always face the earth. The ephemeris information is relayed to the navigator via modulation patterns on the 150- and 400-MHz transmissions, which are never turned off.)
2. There are four stations (in Hawaii, California, Minnesota, and Maine) that "track" the satellite signals at every opportunity. By "track" we mean that the stations measure the frequency of the satellite signal at 4-s intervals. After the satellite has set (typically, 17 min elapse from rise to set), the measurements are transmitted to a central computing facility where all measurements from all tracking stations for each satellite are accumulated. At least once a day they are used in a large computing program to
  - a. Determine a contemporary orbit specification for the satellite and prepare an ephemeris of the satellite for the next 16 h.
  - b. Compute the necessary corrections to the satellite clock to compensate for the predictable part of the oscillator drift - typically, several parts in  $10^{11}$ /day.
  - c. Calibrate all tracking station oscillators and clocks relative to a common standard.

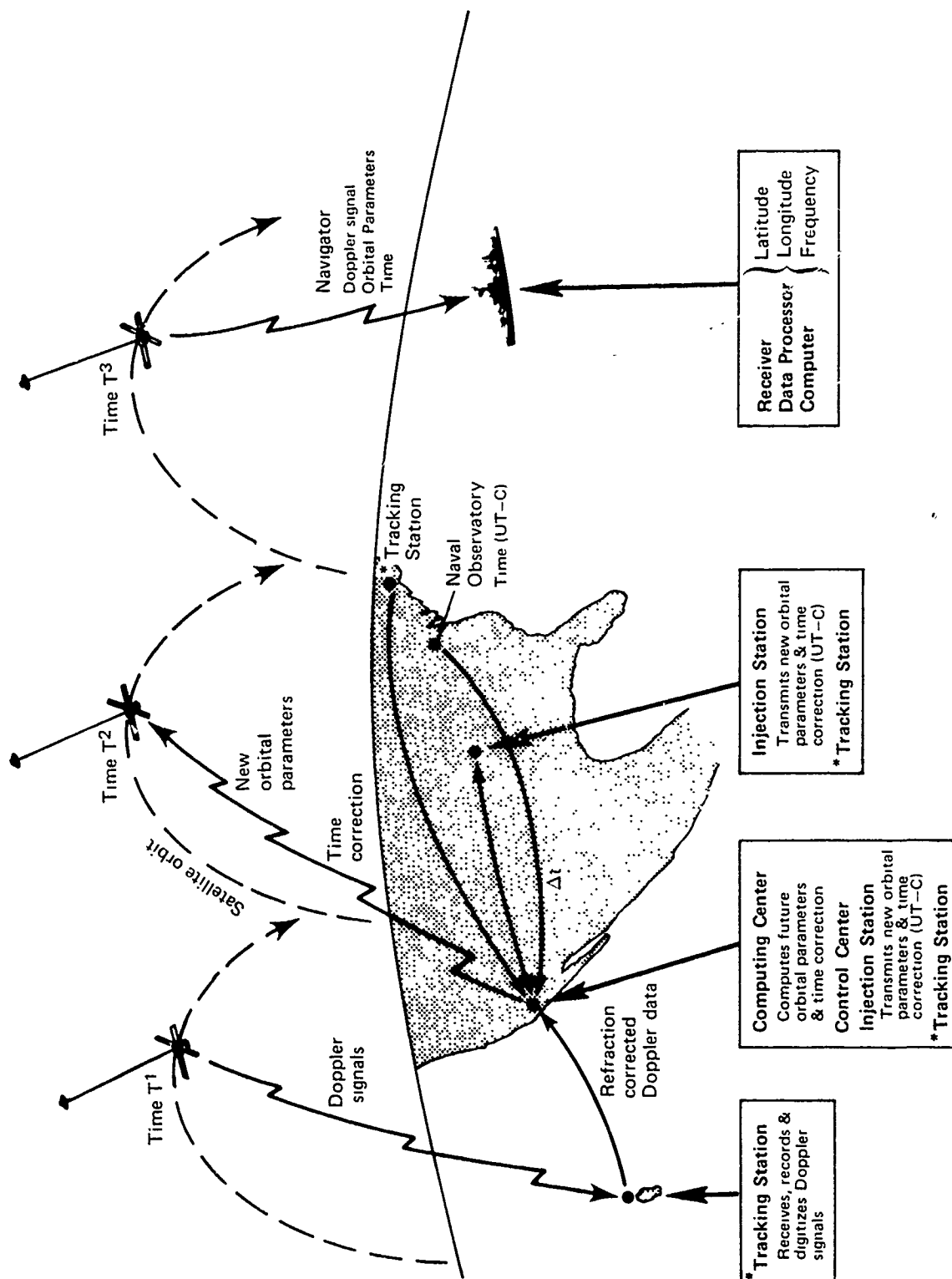


Fig. 1 Schematic of the Transit System.

The ephemeris prediction and satellite clock correction information is then transmitted back to one of the three "injection" sites. (One station actually performs the injection while a second provides backup in case of equipment failure.)

3. The injection station inserts this ephemeris into the satellite memory, writing over one that is about to expire. Typically, injections into the satellite occur at 12-h intervals. (Every satellite is visible at every station at least once every 12 h. The satellite memory has sufficient storage to contain a 16-h ephemeris.)
4. In using the satellite to determine his position, a navigator both measures the received frequency at discrete intervals and demodulates the satellite carrier to recover the satellite ephemerides. With his frequency measurements, the orbit, and his own motion, he can compute his position. It is not simple, nor amenable to hand computation. However it is easily programmed for a small digital computer.

These then are the basic elements of the system. We have omitted, in the interest of brevity, a number of details that the interested reader may find in the references and bibliography. We have provided an annotated guide to the more important sources in the appendix.

### 3. CURRENT STATUS

The May 1975 constellation of satellites is shown in Table 1, together with their launch dates. All have been in near-continuous service. (Recently a sixth satellite died after 6½ years of use.) As it was not anticipated that the satellites would last so long, there are 12 in storage awaiting the demise of those currently in orbit.

Table 1  
Satellites in Service, May 1975

Satellite International ID	Satellite Broadcast ID	a (km)	$\epsilon$	i (deg)	$\Omega^*$ (deg)	Date Launched
1967-34a	30120	7441	0.002	90.2	17.7	14 Apr 1967
1967-48a	30130	7464	0.001	89.6	322.7	18 May 1967
1967-92a	30140	7454	0.004	89.2	343.3	25 Sep 1966
1970-67a	30190	7465	0.019	90.9	249.1	27 Aug 1970
1973-81a	30200	7399	0.016	90.2	117.9	29 Oct 1973

\*Epoch: Day 121, 1975

Ideally, the orbits should all be exactly polar ( $i = 90$  degrees) and should have their nodal longitudes ( $\Omega$ ) uniformly distributed around the equatorial plane. Because we do not have perfect launch vehicles, this ideal is not realized.

#### 4. OPPORTUNITIES FOR OBTAINING A FIX

Since the earth's rotation carries a user under an orbit limb every 12 h and the satellite period ( $\sim 107$  min) is short compared with a day, there are several opportunities for obtaining a fix when a navigator is near the orbit plane. Typically, there will be several fixes spaced approximately 107 min apart and then a long gap of 8 to 10 h. The sequence will then repeat. As this is true for any of the five satellites, fixes are available roughly every  $1\frac{1}{2}$  h more frequently for a navigator near the poles and less frequently near the equator.

## 5. ACCURACY AND QUALITY CONTROL

The operating agency, the Navy Astronautics Group, whose headquarters are located at Point Mugu, CA, is responsible for maintaining the satellite system on a continuing basis. This group of roughly 250 people maintains rigorous quality control procedures over the accuracy of its computations and the performance of the satellites, the oscillator drift, signal levels, the ground station hardware, satellite and station clocks, etc. Some measure of their accomplishment in operating the system can be obtained from the following statistics (from Ref. 5):

"...From 11 October 1968 until 28 February 1975, there were 22,871 memory reloads, 5 of which could not be verified as having properly refreshed the satellite memory. For three of these five injections there is a possibility that the message being transmitted was good but its validity could not be absolutely determined. [Satellite reliability is defined as the percentage of in-service time the satellites were transmitting valid navigation information.] Between 11 October 1968 and 18 February 1975 the NNSS satellite reliability was 99.988%. Between 22 January 1973 and 28 February 1975, in-service reliability was 100%... ."

Several other organizations independently certify the accuracy (and integrity) of the system on a continuous basis. For example, at APL we have a navigation receiver that monitors all satellites at every available opportunity. We have been doing this since 1963. The data are used to navigate the site using the ephemeris obtained from the satellite. Figure 2 shows typical results.

The satellites are also routinely monitored by the TRANET System of tracking sites. (These tracking stations were originally installed to support the development phase of Transit [1958-1963] but currently play no role in its normal operation.) In addition, the Transit System data provide enough redundancy so that the system is, by and large, self-checking. For example, no data are admitted to the orbit computation unless they are "consistent" with the previously obtained orbit. Consistent means that a navigation result produced with the data and the extrapolated ephemeris

---

Ref. 5. Navy Astronautics Group, Pt. Mugu, CA, private communication, March 1975.

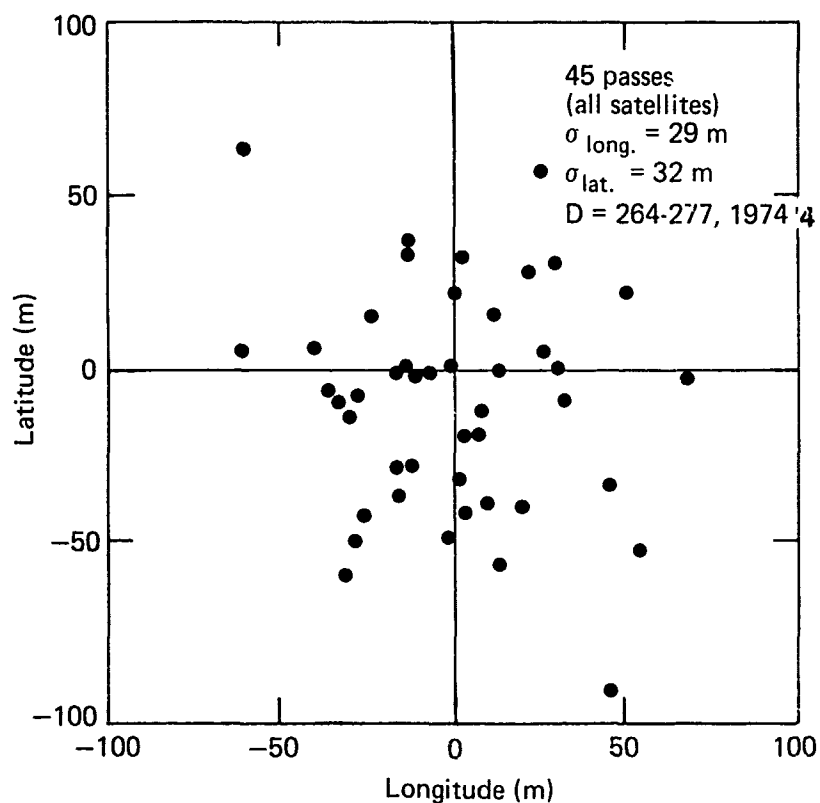


Fig. 2 Transit Surveying Results.

lies within the expected neighborhood of the known station location.

In addition to this criterion, the data must be reasonably free of gaps (data gaps are associated with instrumentation or signal-level problems) and satisfy internal consistency tests on the noise. It suffices to say that the data editing is the most intricate and involved part of the orbit determination computation (Ref. 6).

Anyone with a Transit navigation receiver/computer can measure the internal consistency for himself simply by performing repeated navigations at a fixed site.

Ref. 6. H. D. Black and B. J. Hook, "The Data EDITOR," APL/JHU TG 756, 1966.



## 6. BROADCAST EPHEMERIDES

The ephemerides broadcast from the satellite are generated by a least-squares fitting of the Doppler measurements to an analytical formulation of the Doppler shift (Ref. 7). The mathematical modeling of the Doppler shift and the equations of motion of the satellite are as precise as we can write and currently include a  $(15 \times 15)$  model of the geopotential, direct luni-solar perturbations, radiation pressure, polar motion, and the Jacchia model of the upper-atmosphere density (as part of the drag model) (Refs. 8, 9, and 10).

For insertion in the satellite, the precision ephemeris is factored into a precessing-ellipse formulation plus corrections to this approximate form. The combination of the two parts retains all the precision of the original ephemeris. A word-length restriction in the satellite memory forces us, as a final step, to round the ephemeris to the nearest 10 m. This numerical, random noise source is then  $\pm 5$  m with a period of 2 or 4 min. This is a minor error source (see Table 6, Section 11) for a moving navigator. The fixed navigator (surveyor) uses multiple passes from several satellites and, as a consequence, many independent samples of this noise; nevertheless, it is not a negligible error source for users interested in the highest possible precision.

A larger navigation error source arises from incorrectly modeled drag and radiation pressure acting on the satellite over the extrapolation interval. Of these two, drag is the more serious, in spite of the fact that drag is usually far smaller than is the radiation pressure. This is because drag always opposes the along-orbit motion; thus its effect is cumulative. Drag determines how frequently the satellite orbit is redetermined. The normal mode of operation dictates that there be no measurable secular

---

Ref. 7. H. D. Black, "Doppler Tracking of Near-earth Satellites," APL/JHU TG 1031, 1968.

Ref. 8. L. G. Jacchia, "Static Diffusion Models of the Upper Atmosphere with Empirical Temperature Profiles," Smithsonian Contributions to Astrophysics, Vol. 8, No. 9, pp. 215-257, 1965.

Ref. 9. B. B. Holland, J. A. Yingling, and M. A. Walko, "The Second Generation Integration Routine (IGC)," APL/JHU TG 466 (Rev.), 1970.

Ref. 10. A. Eisner, "Atmospheric Density Studies," APL/JHU TG 951, 1967.

growth in the satellite position error over the extrapolation interval. The satellite position error is then dominated by geopotential modeling errors. Currently, near the minimum of the solar ultraviolet cycle, this means that the satellite orbit must be re-determined daily.

As we currently understand the Transit System, there is no convincing reason for a user to compute ephemerides independently in hopes of achieving higher precision results. The error sources that can be reduced by independent orbit determination (the drag-induced errors over the prediction interval,\* the word-length-restriction error, the geopotential model errors) can also be reduced simply by using a larger data population than would otherwise be necessary and all satellites. Moreover, the process can proceed in real time in the field. As each fix is obtained, the mean of all fixes is updated. When this mean stabilizes with a variation less than (say) several meters, it is time to stop as more data will not help. The March 1975 error budget and satellite constellation dictate that this will require 1 to 3 days and 20 to 80 passes (see Fig. 2).

The significant fact is that all orbit determination schemes — whatever the claims for their precision — are limited by the same things, errors arising in

1. The instrumentation used to acquire the data;
2. The coordinates of the pole;
3. Ignoring the attitude motion of the satellite;
4. Higher order propagation (ionospheric) effects; and
5. The instability of the satellite oscillator.

All are on the order of a meter and, moreover, probably put long-term correlation (compared with the pass length) in the navigation error. We will have more to say on these problems in a later section.

---

\*The effects of drag (along-track bias) on the navigation result change sign for north- and south-going passes. Consequently the drag error is largely cancelled.

## 7. SATELLITE CLOCK

By today's standards, the satellite clock need not be very accurate. This statement is true because system errors caused by timing (bias) errors are never greater than  $|V_{SAT}| \cdot \Delta t$ ,  $|V_{SAT}| \approx 7000$  m/s. If then the time bias  $|\Delta t|$  is less than  $10^{-4}$  s, navigation errors caused by clock errors will be less than 1 m. Generally the clock-associated errors are less than half this amount (Fig. 3). The clock is maintained as a routine part of the normal system operation by the Navy Astronautics Group. The technique used for "minding" (and utilizing) the clock is described in Ref. 11. The primary time standard for Transit is UTC.

Periodically, at 6- to 12-month intervals, a portable cesium clock (and frequency standard) visits all four tracking sites, after having been set at the Naval Observatory. Secondary cesium standards are maintained at all sites. Some recent results of these visits are shown in Table 2. We have omitted the Hawaii

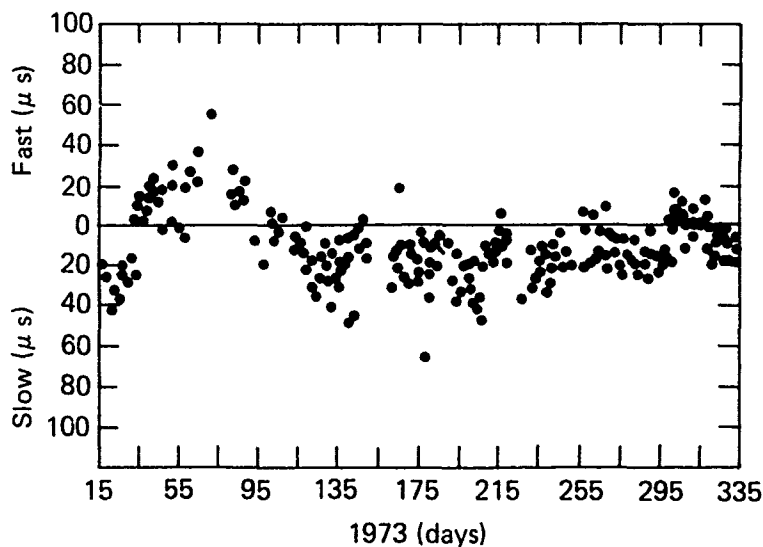


Fig. 3 Satellite 1967-92a Clock Error During 1973.

Ref. 11. C. Marvin, "The Operation of the Satellite Clock Control System," APL/JHU TG 523, 1963.

Table 2  
Time and Frequency Errors at the  
Transit Tracking Sites

Station	Date	Clock Error ( $\mu$ s)	Frequency Bias  (parts in $10^{12}$ )
Maine	Jan 72	16	--
	Apr 72	7	--
	Oct 72	9	1
	Apr 73	3	1
	Sep 74	15	2
Minnesota	Dec 71	10	4
	May 72	25	--
	Jun 73	17	7
	May 74	18	4
	Nov 74	40	25
California	Dec 71	1	--
	May 72	11	3
	May 73	1	--
	May 74	1	3
	Nov 74	40	--

Station because it is operated in conjunction with a Naval Observatory time and frequency standard. The largest time bias measured at the Hawaii Station is 5  $\mu$ s.

The Naval Surface Weapons Center routinely evaluates the satellite clock error using data obtained from the TRANET System, i.e., a system of stations that are independent of the stations used in setting the satellite clocks. Figure 3 shows their data on Satellite 1967-92a for 1973. From the data, the satellite clock error is generally less than 50  $\mu$ s. The system has maintained this precision since 1968.

## 8. GEOPOTENTIAL MODEL

There have been only four geopotential models used in the Transit System since the beginning. Listings of coefficients and geoid maps (for the last two) can be found in Ref. 7. The development of these models was by and large the work of W. H. Guier, R. R. Newton, S. M. Yionoulis, and F. T. Heuring. We have compiled Table 3 to illustrate the precision with which these models will determine a satellite orbit.

Table 3  
Precision of Satellite Orbit

Geopotential Model	Orbit* Precision (m)	Comments
APL 1.0	100-150	Zonals and sectorials through (4,4). Used from December 1963 through December 1965.
2.0		Zonals and sectorials through (6,6). Used from January 1965 through February 1966.
3.5	75-110	Zonals and sectorials through (8,8) plus a few resonant terms of order 13 and 14 (Ref. 12). Used February 1966 through June 1968.
4.5	15-20	Complete through degree and order 11 plus most terms through (15,15) (226 coefficients). Used June 1968 through July 1975. (Ref. 7)
WGS-72	5-10	Coefficient set complete through degree and order 19, zonals through degree 24, and additional resonance terms through order 27 (479 coefficients). (Ref. 13)

\*Satellite altitude, 1100 km.

Ref. 12. W. H. Guier and R. R. Newton, "The Earth's Gravity Field as Deduced from the Doppler Tracking of Five Satellites," J. Geophys. Res., Vol. 70, No. 18, pp. 4613-4626, 1965.

Ref. 13. T. O. Seppelin, "The Department of Defense World Geodetic System 1972," The Canadian Surveyor, Vol. 28, No. 5, pp. 496-506, Ottawa, Canada, December 1974.

As shown in Fig. 4, these model accuracies can be placed in a common frame of reference using an analysis developed by Holland et al. (Ref. 14), and Guier and Newton (Ref. 15).

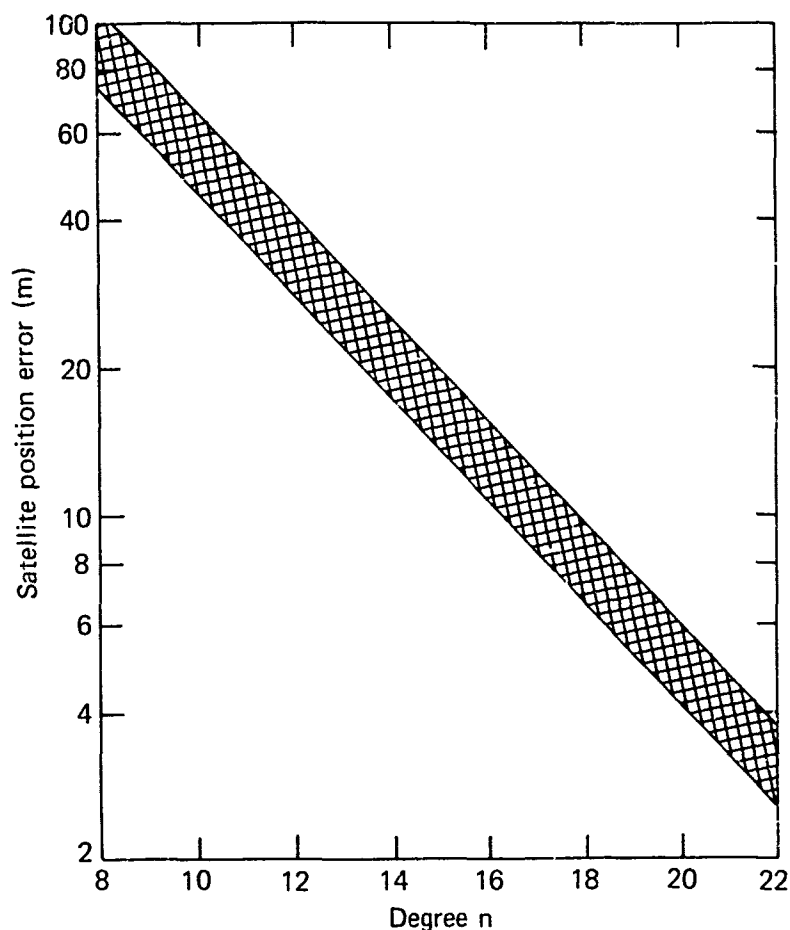


Fig. 4 Satellite Position Error Caused by Truncating the Geopotential Coefficients at Degree  $n$ , Satellite at 1100-km Altitude.

Ref. 14. B. B. Holland et al., "Approaching Geodesy in the 1970's," APL/JHU TG 1055, 1969.

Ref. 15. W. H. Guier and R. R. Newton, "Status of the Geodetic Analysis Program at the Applied Physics Laboratory," APL/JHU TG 652, 1965.

## 9. AT-SEA NAVIGATION

Ship navigation using Doppler measurements is, in principle, exactly the same as fixed navigation (surveying), but in practice, it is somewhat different. The reason is that the surveyor knows his velocity with great precision; he is fixed relative to the earth. The at-sea navigator must have independent knowledge of his velocity. Moreover, he has a noise source that is not present in the surveyor's computation: the erratic motion of his antenna caused by sea motion. (Since the doppler shift is a component of relative velocity, any unaccounted terms in the antenna motion are noise/error sources.)

Several things can ameliorate these problems, but first we should explain the net effect: Following Newton (Ref. 16), we derive that the principal effect of the navigator's velocity uncertainty is given by

$$\delta = \left\{ \frac{r^2 - 2r \cos(\Omega) + 1}{\cos^{-1}(r \cos \Omega)^{-1} \cdot r^2 \sin(\Omega)} \right\} \cdot |\delta V| T_D,$$

where

$r$  is the geocentric distance to the satellite in units of the earth radius,

$\Omega$  is the longitude difference between the navigator and satellite when the satellite is at closest approach,

$|\delta V|$  is the north component of the navigator's velocity uncertainty, and

$T_D$  is the pass duration.

The quantity in brackets lies between 0.5 and 1.1 for pass elevations lying between  $15^\circ$  and  $75^\circ$ . Over this same elevation interval, it has a mean value of 0.65.

---

Ref. 16. R. R. Newton, "The U.S. Navy Doppler Geodetic System and Its Observational Accuracy," Phil. Trans. Roy. Soc. London, Vol. A262, pp. 50-66, 1967.

This equation is the basis of a handy rule of thumb that is (almost) intuitively obvious:

$$\delta \approx K |\delta V| \cdot T_D .$$

For  $K = 1$ , this is just the dead-reckoning error at the end of the pass. For  $T_D = 1000$  s (typically),  $|\delta V| = 1$  kt (0.5 m/s), and with  $K$  slightly less than unity, we get 350 m.

As a result (cf. Table 6), the error budget of the usual at-sea navigator is dominated by the velocity-uncertainty effect. This is, of course, not true for ships having high-quality inertial or Doppler sonar systems. A surprising fact is that an error in the north-velocity component causes an error in the east-west position. For more elaborate discussions on this subject, see Newton (Ref. 16) and Sluiter (Ref. 17).

The usual at-sea navigator could not care less if we provide enough precision in the fix for him to tell his bow from his stern. As a consequence, the 100- to 200-m error that comes from, say, a 1/4- to 1/2-kt speed error is of little concern. On the other hand, there are some users for whom this is a serious deficiency, such as the people using Transit in offshore oil exploration and oceanographers using the system for at-sea magnetic anomaly surveys. Some of these users have elaborate ship-velocity instrumentation (Doppler sonar combined with precision inertial equipment); some do not. The following remarks hopefully will help both types: Table 4, from Ref. 18, indicates that many, perhaps most, ships have natural roll and pitch periods lying between 5 and 20 s. Manning comments

"...the natural period of pitching is usually between one-third and two-thirds the natural period of roll.... Since ocean waves are not usually of constant period for any sustained length of time, and since the longitudinal inertia of a ship is very great in comparison to its transverse inertia, it has been found that a ship pitches in its own natural period a much greater part of the time than it rolls in its own natural period. In other words, forced pitching occurs

---

Ref. 17. P. C. Sluiter, "Relative Weighting of Satellite Fixes," Proceedings of a Symposium on Marine Geodesy, New Orleans, LA, p. 151 et seq., 3-5 November 1969.

Ref. 18. G. C. Manning, "The Motion of Ships Among Waves," Chapter 1, Principles of Naval Architecture, II, H. E. Russell and L. B. Chapman, eds., Society of Naval Architects and Marine Engineers, p. 45, 1942.



Table 4  
Data on Roll and Pitch Periods of Ships

Ship	Type	Length (ft)	Breadth (ft)	Draft (ft)	Displace- ment (tons)	Period of Pitch (s)	Period of Roll (s)
Montcalm	Passenger	546	70.00	24.50	19,670	7.20	17.00
London Mariner	Express cargo	450	57.75	27.00	14,660	6.80	9.00
San Gerardo	Tanker	530	64.00	30.75	25,700	10.20	10.00
San Tirso	Tanker	420	54.50	26.75	13,700	9.80	9.00
Aquitania	Express passenger	---	---	---	---	8.00	---
Lusitania	Express passenger	---	---	---	---	8.00	---
Ship A (laden)	Well-deck collier	230	33.10	16.00	2,450	6.10	9.65
Ship A (in ballast)	Well-deck collier	---	---	---	---	6.50	6.60
Ship B (laden)	Raised-quarter-deck collier	235	36.00	14.60	2,620	5.10	7.80
Ship E (laden)	Three-island collier	240	36.50	18.60	3,500	6.30	12.30
Ship E (in ballast)	Three-island collier	---	---	---	---	4.90	7.80
Ship F (in ballast)	Raised-quarter-deck collier	245	37.00	18.30	3,520	4.90	7.60
Ship G (laden)	Arch collier	245	37.20	17.90	3,620	5.03	9.00

Source: Ref. 18.

less frequently than forced rolling. Experience has shown that the period of pitching at sea is usually the ship's natural period...."

Receivers for at-sea use should be chosen to avoid the 5- to 20-s period for the Doppler-counting interval, particularly if the necessary clinometer data are not provided to correct for the roll-pitch effects in the navigation computation.

It would be helpful if we could determine a ship's velocity and position from a single pass of data. We have been unable to satisfy ourselves that such a computation is practical. For a satellite passing directly overhead (navigator in the orbit plane), the measured Doppler shift is very insensitive to the navigator's velocity-east component. Were it not for the earth's rotation, we could say it was "immune" to velocity-east. As a result, attempts to determine both components of the navigator's position and velocity, using unaided Doppler data from a single satellite, do not appear practical. For passes having elevation angles that exceed  $15^\circ$ , the longitude and velocity-north are strongly coupled and the effects of velocity-east are practically negligible.

## 10. PLANNED IMPROVEMENTS

Beginning in August 1975, a number of improvements are planned for the Transit System. Some are designed specifically to automate system operation and thereby cut the operation and maintenance costs. These will not be described here; however, reduction in operating costs should have a beneficial effect on the system's life expectancy.

The changes of real interest to users are those that improve the system precision (internal consistency); they are, first, in the orbit-determining software and, second, in the hardware.

### 10.1 Orbit-determination Software Changes

We are:

1. Replacing the APL 4.5 geopotential model with the WGS-72 model;
2. Consistent with item 1, updating the GM (gravitational constant times the earth mass) from its currently used value of  $398601.5 \pm 0.6 \text{ km}^3/\text{s}^2$  (Ref. 19) to the more recent determination — including the atmosphere — of  $398600.8 \pm 0., \text{ km}^3/\text{s}^2$  (Ref. 20). We are leaving the speed of light unchanged at 299,792.5 km/s;
3. Altering the coordinates of the four tracking sites to bring greater internal consistency to the overall network. These changes are given in Table 5;

---

Ref. 19. W. L. Sjogren, "The Ranger III Flight Path and Its Determination from Tracking Data," JPL Technical Report, 32563, 22, Jet Propulsion Laboratory, Pasadena, CA, 15 September 1965.

Ref. 20. P. B. Esposito and S. K. Wong, "Geocentric Gravitational Constant Determined from Mariner 9 Radio Tracking Data," Jet Propulsion Laboratory, Pasadena, CA, 1972. (Paper presented at the International Symposium on Earth Gravity Models and Related Problems, St. Louis, MO, 16-18 August 1972.)

Table 5  
Changes to the Transit Stations' Coordinates  
(in meters)

Station Location	Latitude	Longitude	Radius
Maine (311)	-0.3	-4.6	+0.6
Minnesota (321)	+2.0	+3.2	+5.4
California (330)	+1.8	0.0	+0.3
Hawaii (340)	+6.1	+0.6	-4.5
Mean Change	+2.4	-0.2	+0.5

Note: To be implemented in the fall of 1975.

4. Implementing the (main) body-tide perturbation on the satellite. This force had previously been neglected; and
5. Introducing an improved model of the radiation pressure forces.

The changes are scheduled for implementation beginning in August 1975. No changes are required of any user. We do suggest that the numerical integrity (consistency) of the older navigation programs should be reexamined to ascertain that they do not introduce spurious numerical noise as large as 1/2 m. We are leaving the angular velocity of the earth unchanged at  $7.29211585 \times 10^{-5}$  rad/mean-solar-s because there is nothing to be gained by changing it.

#### 10.2 Hardware Changes

A new type satellite is being introduced into the constellation in late 1975. Of principal interest is the DISCOS\* device (Ref. 21) and a high precision clock. These improvements will be discussed in detail.

---

\*Disturbance Compensation System, a device that accelerates or decelerates the satellite to compensate for the effects of drag and radiation pressure.

Ref. 21. Staff of the Space Department, The Johns Hopkins University Applied Physics Laboratory, and the Staff of the Guidance and Control Laboratory, Stanford University, "A Satellite Freed of All But Gravitational Forces: TRIAD I," J. Spacecraft and Rockets, Vol. 11, No. 9, pp. 637-644, 1974.

## 11. WGS-72 GEOPOTENTIAL MODEL

The single-pass error budget for a surveyor is given in Table 6 both "before" and "after" the introduction of WGS-72.

Table 6  
Transit System - Surveyor's Single-Pass Error Budget

	Meters
1. Uncorrected propagation effects (3rd order ionospheric and neglected tropospheric effects)	1-5
2. Instrumentation (navigator satellite oscillator phase jitter)	1-6 (See note)
3. Geodesy (uncertainty in the geopotential model)	15-20 (APL 4.5)
4. Incorrectly modeled surface forces (secular error growth due to incorrect period, drag and radiation pressure)	5-10 (WGS-72)
5. Unmodeled UT1-UTC effects and incorrect coordinates of the pole	10-25
6. Ephemeris rounding error (last digit of ephemeris is rounded)	1
rss	5
	19-33 (APL 4.5)
	12-28 (WGS-72)

Note: We have some data that indicate that the Geociever oscillator/Transit satellite oscillator contribution is appreciably less than 1 m (Ref. 22). This performance is not unique to Geociever but characteristic of the more modern receivers. For the older SRN-9, the contribution is 3 to 6 m (Ref. 7).

Ref. 22. B. B. Holland, "Uses of Geociever as a Geodetic Instrument," Proceedings of COSPAR XIII, Madrid, Spain, 10-24 May 1972, Akademie-Verlag, Berlin, p. 71, 1973.

It is clear that implementing WGS-72 will not have a striking effect on the single-pass error budget; for the existing constellation, the budget will continue to be dominated by items 3 and 4 from Table 6. We decided to implement WGS-72 for the following reasons:

1. There is a beneficial side effect even for existing satellites. Reducing the correlated (geodesy) errors gives improved access to the orbit state vector, and therefore the orbit extrapolation errors are diminished. (Since it is not clear how this improvement will be utilized, we have not reflected this improvement in the error budget.)
2. We found that if we restructured the computation of the force terms in the satellite equations of motion, we could implement WGS-72 with a minimal increase in computing time.
3. The integrity of any previously derived Transit results would be preserved; the discontinuity in results (Table 5) when the system is changed from one geopotential model to another would lie within the previously advertised 1- to 5-m precision (Ref. 23) over most of the western hemisphere. In the Indian Ocean the differences reach a peak of 15 m. We were careful to preserve the longitude reference of the coordinate system which is implicit in the station coordinates (Ref. 24).
4. Of some real significance for the new satellite is that, with the single-axis DISCOS system, there is good reason to believe that drag errors will be reduced by a factor of 10 (see Section 16). If this turns out to be true, then the largest error sources for fixed site navigation will be appreciably reduced. For the DISCOS-equipped satellite, the single-pass error budget will become 7 to 13 m; with the combination of DISCOS and WGS-72, we will have halved the current 19- to 33-m error budget.

---

Ref. 23. V. L. Pisacane, B. B. Holland, and H. D. Black, "Recent (1973) Improvements in the Navy Navigation Satellite System," Navigation, Journal of the Institute of Navigation, Vol. 20, pp. 224-229, 1973.

Ref. 24. G. Gebel and B. Matthews, "Navigation at the Prime Meridian," Navigation, Vol. 8, pp. 141-146, 1971. (Note: A recent redetermination of the connection between the astronomical meridian and the geodetic meridian replaces the 5.64" [given in this paper] with 5.69"  $\pm$  0.17".)

There is another reason, a very human reason, for implementing WGS-72: Describing coordinate systems, geopotential models, datums, etc., is a very tedious job. It is a great convenience to simply say, "the Transit System is on WGS-XX."

#### 11.1 Comparison with Other Geopotential Models

Since certain aspects of WGS-72 are classified, we are not free to describe it in great detail. This fact does not impugn its usefulness in deriving positional information. We have performed a number of experiments comparing orbits derived with WGS-72 with those obtained using other geopotential models, and leaving all other parts of the process(ors) unchanged.

The orbit error is most conveniently represented as station position error (at known sites) resolved in orbit-related coordinates (Ref. 7).

We determined a single-state vector (utilizing all data collected at the four tracking sites over two 4-day periods) employing WGS-72. After the orbit determination computation was complete, we then treated each pass of data independently in a station navigation computation. The navigation "errors" become, then, a measure of the internal consistency of the entire system, the earth model included. We repeated the determination for

1. GEM-1 (Goddard Earth Model 1) (Ref. 25),
2. GEM-6 (Ref. 26),
3. SAO Standard Earth 3 (Ref. 27), and
4. APL Mk 4.5.

The results are summarized in Table 7 where we show the rms errors for both position components. The rms is taken over all data in the 4-day span.

---

Ref. 25. D. E. Smith, F. J. Lerch, and C. A. Wagner, "Gravitations Field Model for the Earth," Proceedings of COSPAR XIII, Madrid, Spain, 10-24 May 1972, Akademie-Verlag, Berlin, 1973.

Ref. 26. F. J. Lerch, J. E. Brownd, J. A. Richardson, and J. S. Reece, "Gravitational Models GEM-5 and GEM-6, 1974," contribution to The National Geodetic Satellite Program Final Report, American Geophysical Union, Washington, DC, to be published August 1975.

Ref. 27. E. M. Gaposchkin, ed., Smithsonian Standard Earth III, Smithsonian Astrophysical Observatory Special Report 353, 1973.

Table 7  
Satellite 1967-48a  
(in meters)

EXP No.	Time Span of Data	WGS-72		GEM-1		GEM-6		APL 4.5		SAO SE 3	
		Along- Orbit Error	Slant- Range Error	Along- Orbit Error	Slant- Range Error	Along- Orbit Error	Slant- Range Error	Along- Orbit Error	Slant- Range Error	Along- Orbit Error	Slant- Range Error
1	D = 40-43, 1974 (51 passes)	6.2	5.6	8.8	6.7	17.6	10.2	11.4	8.4	31.3	21.0
2	D = 44-47, 1974 (55 passes)	4.6	7.0	9.7	8.2	16.4	10.9	8.8	9.7	30.7	24.2
3*	D = 44-47, 1974 (55 passes)	18.3	7.1	28.7	10.5	25.2	13.4	24.6	13.6	37.2	26.6

\*Executed with an ephemeris derived in EXP 1 and extrapolated 96 h into the future.



From the data it would not be difficult to choose WGS-72 in preference to the others. An impressive fact is that GEM-1 was determined from optical data, whereas WGS-72 (and APL 4.5) were determined using a large population of Doppler data, in particular data from the Transit satellites.

As a part of these tests, it was necessary to extrapolate the ephemeris 96 h into the future and repeat the navigations. Uncertainty in drag and radiation pressure corrupt the orbit accuracy, but by the same amount for all model cases; the more accurate period determination possible with WGS-72 is clearly apparent. Graphs showing the along-orbit error component are shown in Figs. 5 through 8 for GEM-1, GEM-6, APL 4.5, and SE 3 geodesy, respectively, with the corresponding WGS-72 results for easy comparison. The left half of each figure shows the orbit error during the data span. The right half shows the error during the 4-day prediction. (Operationally, the satellite-borne ephemeris is replaced before the secular growth rises above the geopotential errors.)

We will have more to say about this secular error in Section 15.

As another check on the internal consistency of the system, we repeated EXP 1 of Table 7 using data from three, rather than four, of the available stations. The data from the fourth station (Minnesota) were used to measure the precision of the resulting ephemeris. During the 4-day period, 24 passes from the fourth station were used in a navigation solution. The mean of the 24 fixes differed 0.6 m in latitude and 1.2 m in longitude from our best estimate of the true station position.

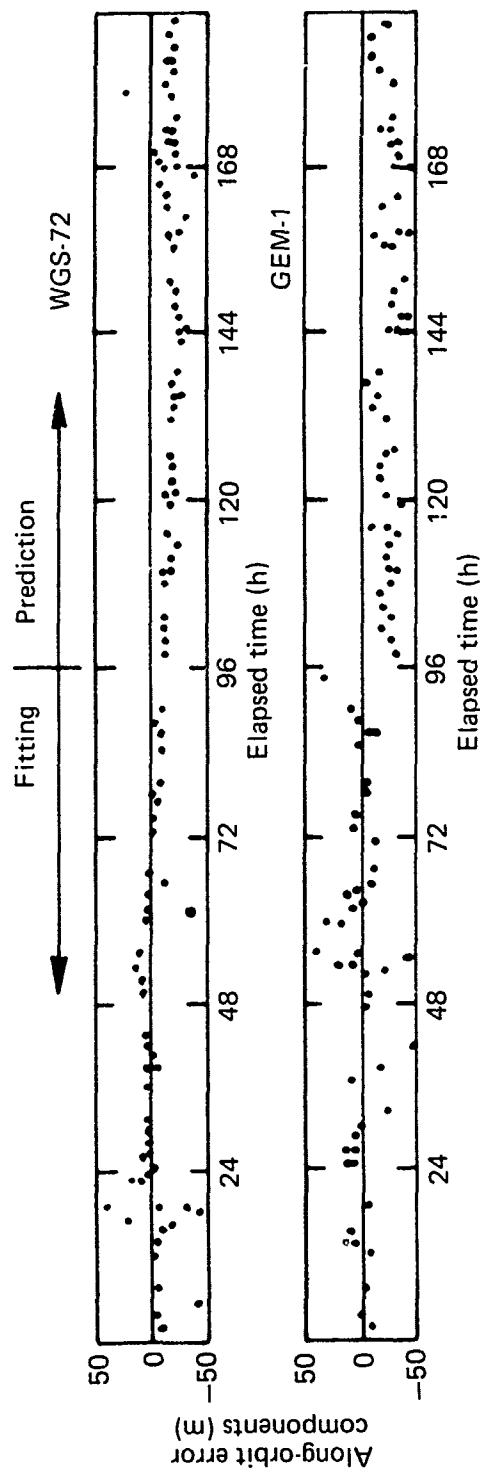


Fig. 5 Orbit Error for WGS-72 and GEM-1 Geopotential Models.

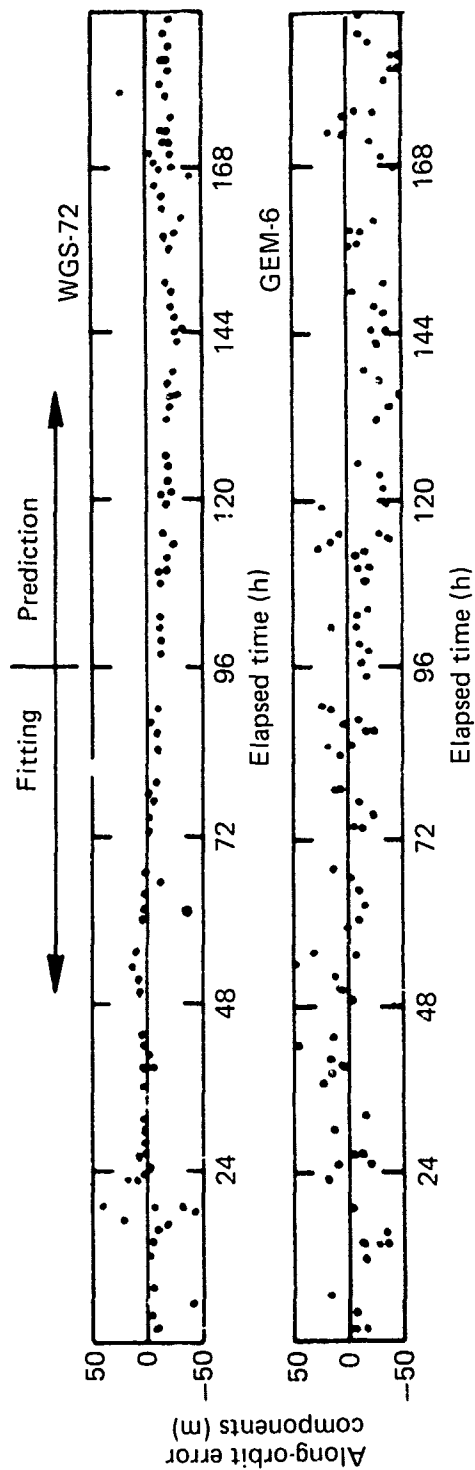


Fig. 6 Orbit Error for WGS-72 and GEM-6 Geopotential Models.

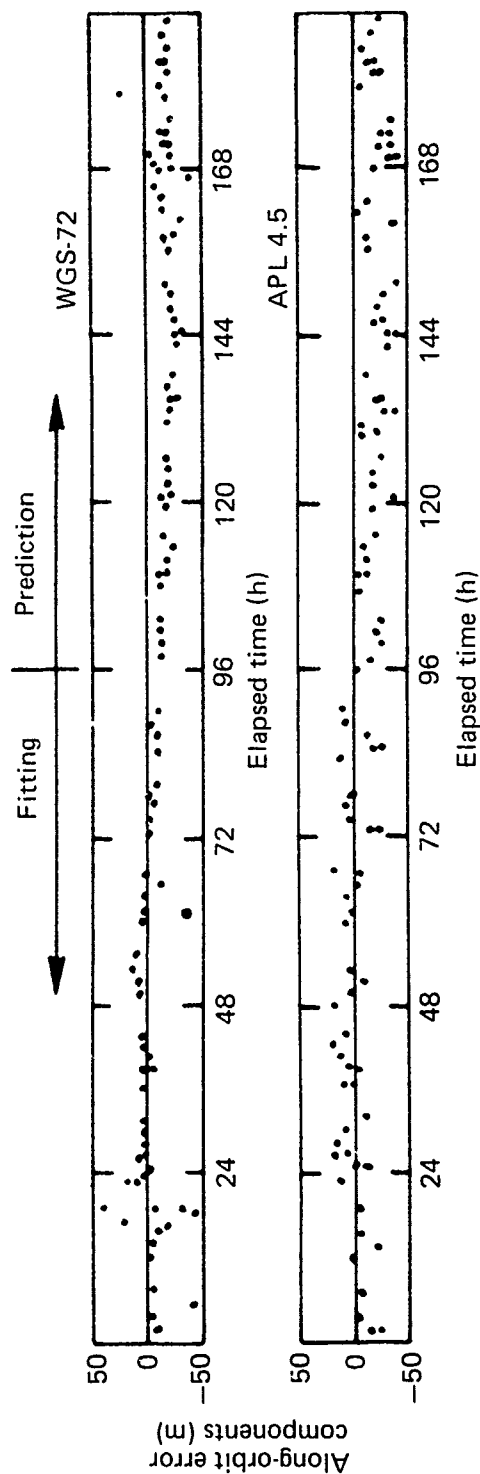


Fig. 7 Orbit Error for WGS-72 and APL 4.5 Geopotential Models.

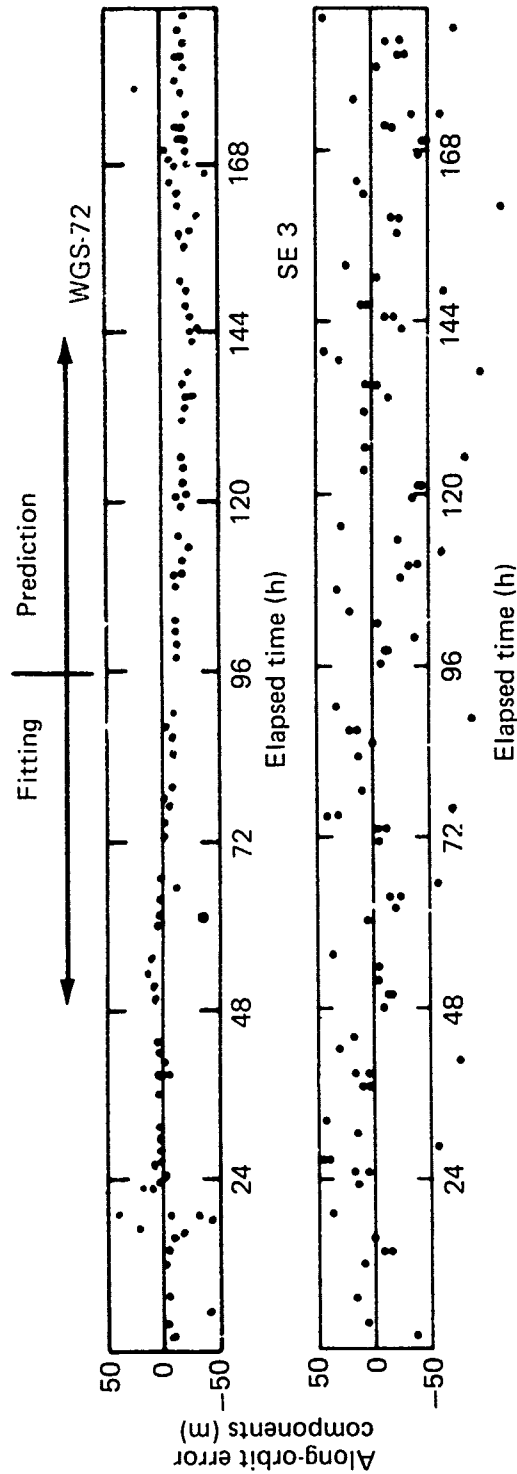


Fig. 8 Orbit Error for WGS-72 and SE 3 Geodesy.

## 12. GEOID HEIGHT CONTOUR MAP

We are frequently asked what geoid map to use with the Transit System. The correct (May 1975) map is shown in Fig. 9; it is rigorously consistent with the currently used (APL 4.5) geopotential. This model is consistent with an ellipsoid having a semi-major axis of

$$a = 6378.137 \text{ km}$$

and a flattening of

$$f = 1/298.25.$$

It is common to report geoid heights relative to various ellipsoids.\* It is not vitally important which ellipsoid is used so long as the map and associated ellipsoid parameters are used in a consistent fashion to construct the distance from the coordinate center (earth mass-center) to the navigator's antenna. It is this distance which is important, rather than any of the mathematical pieces used in constructing it.

There is a 7- to 9-m inconsistency in most navigators' programs, as we originally recommended values of

$$a = 6378.144 \text{ km and}$$

$$f = 1/298.23.$$

Before we describe how to repair this inconsistency it should be pointed out that, as of late 1975, the correct (WGS-72-consistent) value will be

$$a = 6378.135 \text{ km and}$$

$$f = 1/298.26.$$

These values are close enough to the APL-4.5-consistent values that we can correct immediately to the WGS-72 values.

The simplest way to compensate for the inconsistency mentioned above is to alter the antenna height (height above mean sea level). The recipe is as follows:

---

\*Although the geoid height can be computed without directly utilizing the ellipsoid parameters (Bruns' formula), the parameters are usually lurking in the background and therefore implicitly involved.

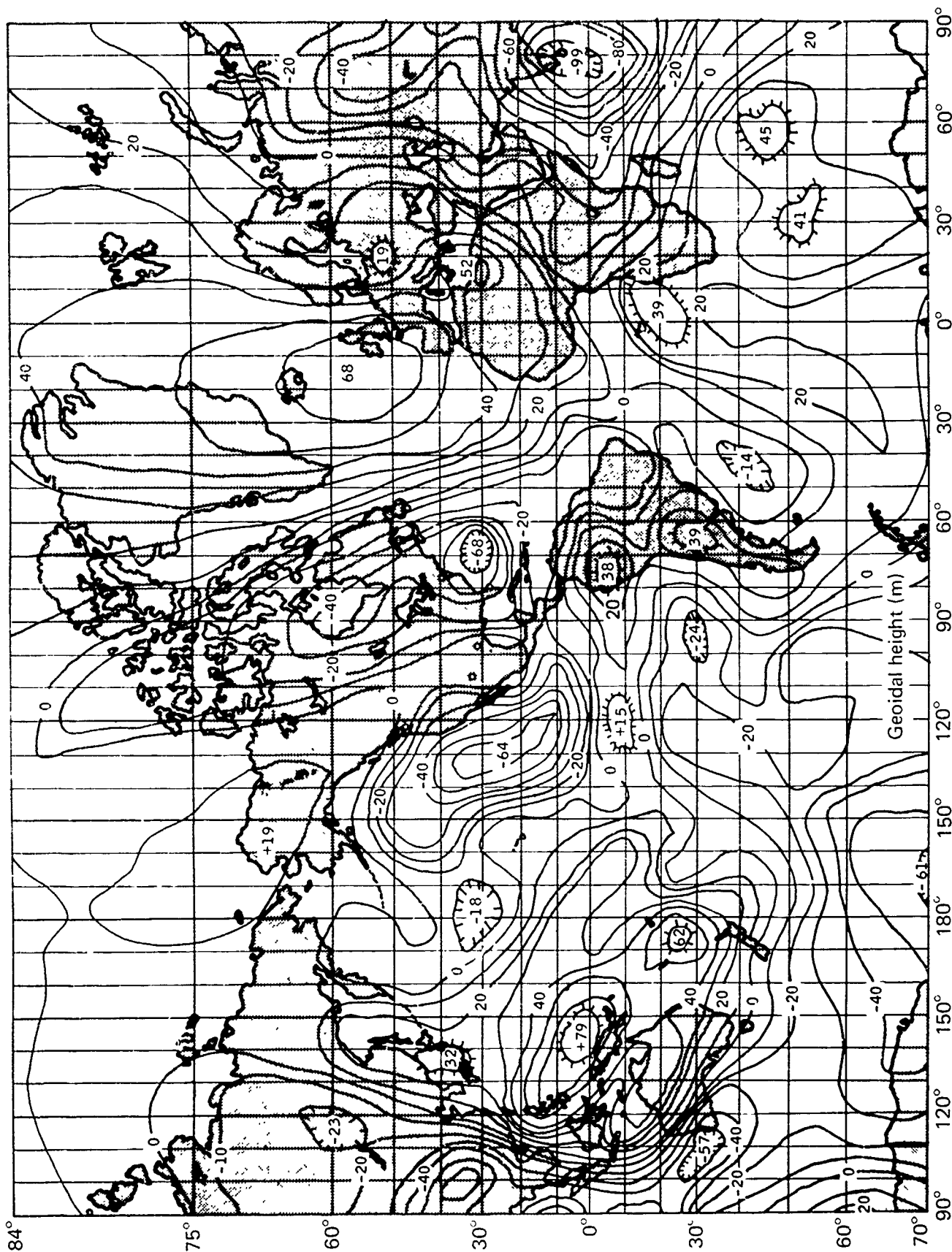


Fig. 9 Transit Geoid Map.

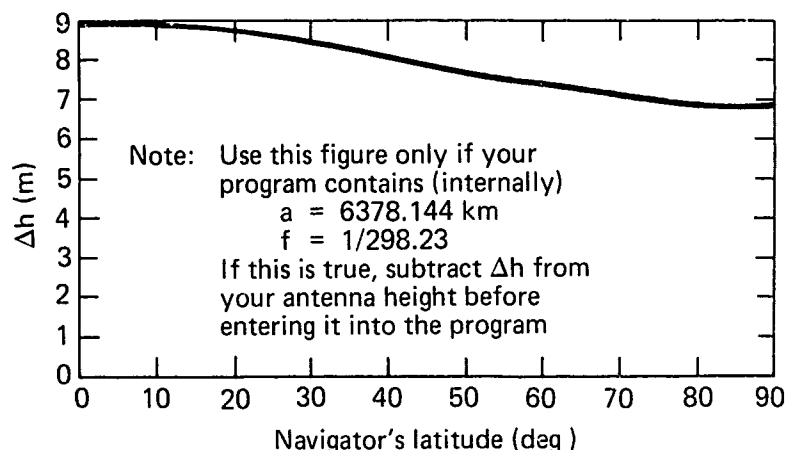


Fig. 10 Antenna Height Compensation.

Replace the antenna height with the antenna height less  $\Delta h$  where  $\Delta h$  is as given in Fig. 10, that is, 7 to 9 m, weakly dependent on the navigator's latitude. The recipe will correct the earth-mass-center/antenna distance for a change in (a, f) from

$$\left\{ \begin{array}{l} a = 6378.144 \\ f = 1/298.23 \end{array} \right\} \quad \text{to} \quad \left\{ \begin{array}{l} a = 6378.135 \\ f = 1/298.26 \end{array} \right\}$$

We hope that most of the navigation programs will be altered internally to the second set of values so that the recipe will not be necessary.

For the surveyor (fixed navigator) another answer to the question "Which geoid map, a, and f?" is that "It really does not matter." The reasonableness of this can be seen from performing the following experiment.

At a fixed site we collect a number of satellite observations (passes) and separate them into two sets; one set is characterized by the satellite passing east of the location and the other by the satellite passing west. Using the antenna height (height above mean sea level), geoid map, and associated ellipsoid constants, individually navigate each data set and plot the mean of the fixes. Change the antenna height (estimate) and iterate until the two mean fixes coincide in longitude. At this point, the distance from the earth-mass-center to the antenna is correct as are the other two position components. Of course, the process can be automated to simultaneously determine all four (latitude, longitude, radius from the earth-mass-center, and frequency bias), but this heuristic (and

practical) technique is useful in understanding why the system does not really require a specific geoid map: The distance from the mass center is determinate if multiple passes are used. Results of such an experiment are shown in Fig. 11.

After the fall of 1975, the at-sea navigator can continue to use the geoid map shown in Fig. 9. An examination of a number of geoid maps (Refs. 26, 28, 29, 30, and 31) and a comparison of them with Fig. 9 have convinced the writers that there is little reason to replace it.

---

Ref. 28. E. M. Gaposchkin, "Earth's Gravity Field to the Eighteenth Degree and Geocentric Coordinates for 104 Stations from Satellite and Terrestrial Data," J. Geophys. Res., Vol. 79, No. 35, pp. 5377-5411, 1974.

Ref. 29. R. H. Rapp, Numerical Results from the Combination of Gravimetric and Satellite Data Using the Principles of Least Squares Collocation, Ohio State University Report No. 200, March 1973.

Ref. 30. R. H. Rapp, Procedures and Results Related to the Direct Determination of Gravity Anomalies from Satellite and Terrestrial Gravity Data, Ohio State University Report No. 211, July 1974.

Ref. 31. S. M. Yionoulis, F. T. Heuring, and W. H. Guier, "A Geopotential Model (APL 5.0-1967) Determined from Satellite Doppler Data at Seven Inclinations," J. Geophys. Res., Vol. 77, No. 20, pp. 3671-3677, 1972.

Station navigations using various station radii

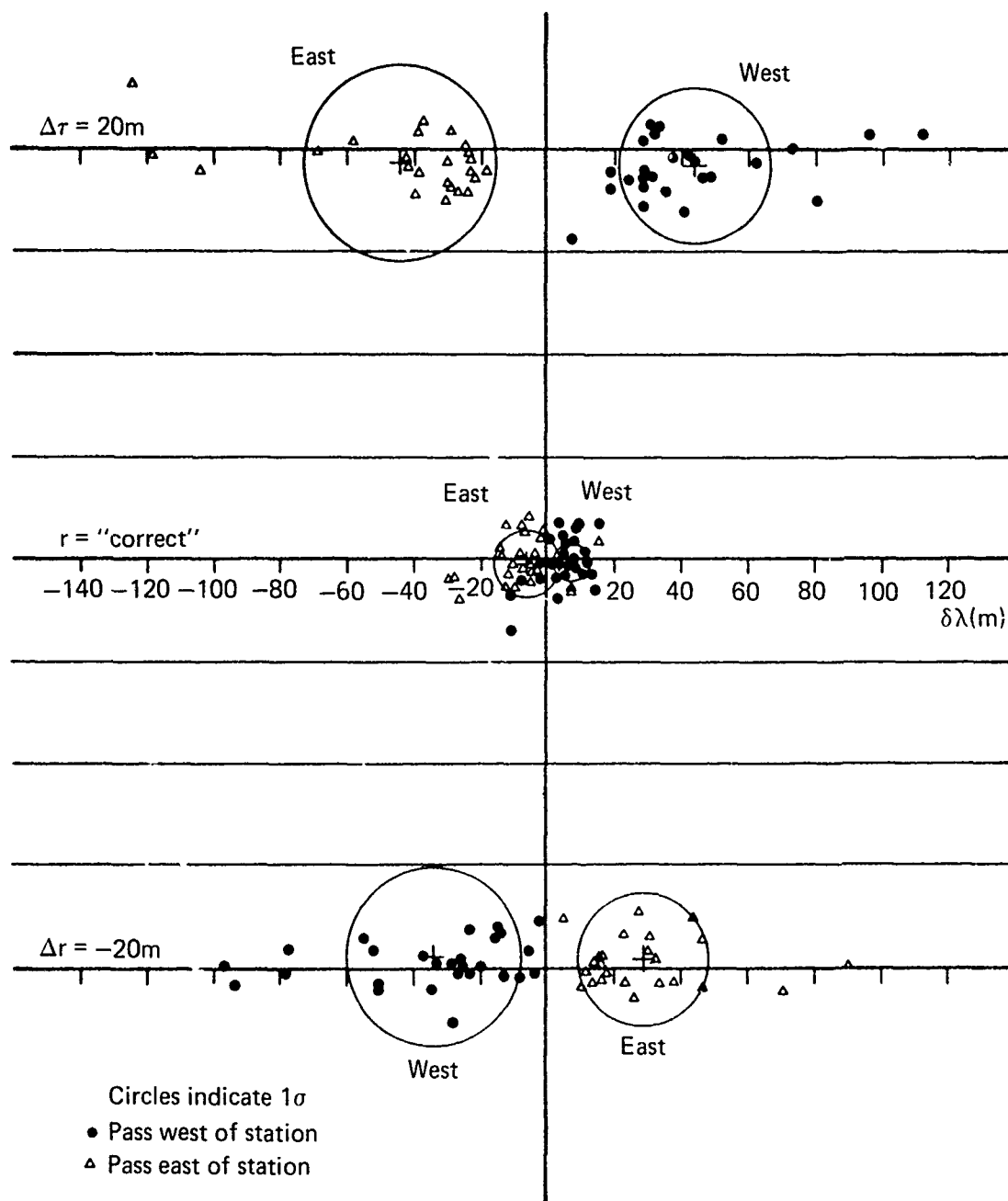


Fig. 11 Determination of the 3-Dimensional Position (all satellites, days 40-43, 1974).



### 13. SUN- AND MOON-INDUCED EARTH BODY-TIDES

To compute the effects of the body-tides (the main semi-diurnal tide) on a near-earth satellite, the geopotential of the earth is augmented by\*

$$V_T = -k \left( \frac{GM_e}{r_d} \right) \left( \frac{M_t}{M_e} \right) \left( \frac{R_0}{r_d} \right)^2 \left( \frac{R_0}{r} \right)^3 P_2(\hat{r}_t \cdot \hat{r}) , \quad (1)$$

and the resulting acceleration of the satellite is

$$-(\nabla V_T) .$$

In this equation

- $k$  is the tidal Love number, 0.336 (Ref. 34), 0.309 (Ref. 35);
- $R_0$  is the radius of the earth;
- $GM_e$  is the product of the gravitational constant and the earth mass ( $398600.8 \text{ km}^3/\text{s}^2 \pm 0.4$ ) (Ref. 20);
- $M_t/M_e$  is the tidal-raising mass/earth mass;

---

\*From Refs. 32 and 33.

Ref. 32. R. R. Newton, "An Observation of the Satellite Perturbation Produced by the Solar Tide," J. Geophys. Res., Vol. 70, pp. 5983-5989, 1965.

Ref. 33. R. R. Newton, "Tidal Numbers and Phases as Deduced from Satellite Orbits," APL/JHU TG 905, 1967.

Ref. 34. R. R. Newton, "A Satellite Determination of Tidal Parameters and Earth Deceleration," Geophys. J. Roy. Astron. Soc., Vol. 14, pp. 505-539, 1968.

Ref. 35. K. Lambeck, A. Cazenave, and G. Balmino, "Solid Earth and Ocean Tides Estimates from Satellite Orbit Analysis," Rev. Geophys. Space Phys., Vol. 12, No. 3, pp. 421-434, 1974.

$\bar{r}$  is the vector from the earth mass-center to the satellite, the point where the potential is to be evaluated,  $\hat{r}$  is the corresponding unit vector, and  $r$ , the length of  $\bar{r}$ ;

$r_d$  is the distance to the tide-raising body; and

$\hat{r}_t$  is the unit vector through the tidal axis, i.e.,

$$\hat{r}_t \triangleq \begin{pmatrix} \cos(\delta) & \cos(\alpha + \alpha_0) \\ \cos(\delta) & \sin(\alpha + \alpha_0) \\ & \sin \delta \end{pmatrix},$$

wherein  $\alpha$ ,  $\delta$  are the instantaneous coordinates (right ascension and declination) of the tide-raising body and  $\alpha_0$  is a small angle, "the tidal phase lag" (0.023 rad for the sun, 0.026 for the moon). Lambeck (Ref. 35) gives 0.0087 for both; and

$P_2(\ )$  is the 2nd order Legendre polynomial =  $\frac{1}{2} [3 \cos^2(\ ) - 1]$ .

A term like Eq. (1) must be included for both the sun and the moon. The amplitude of the sun body-tide is about half that of the moon. Because of the higher frequency associated with the lunar effect, about 13 times the frequency of the solar one, the net effect of the moon on the satellite is about 1/6 that of the sun (Ref. 36).

The determination of the Love numbers (and their associated phases) is currently an active area of research (Refs. 35 and 37). A current goal is to reduce the satellite-determined values for the effects of the ocean tides. Lambeck estimates this effect to be 4 to 9%. Our concern is different; consequently we are using Newton's values which were determined from the Transit satellites and were not corrected for the effect of the (ocean) tides. This is internally consistent with our usage.

---

Ref. 36. R. R. Newton, "Applied Ancient Astronomy," APL Technical Digest, Vol. 12, No. 1, pp. 11-20, 1973.

Ref. 37. B. C. Douglas, S. M. Klosko, J. G. Marsh, and R. C. Williamson, "Tidal Perturbations on the Orbits of GEOS-1 and GEOS-2," NASA Report X553-72-475, 1972.

The principal effect of the body-tide perturbation is an oscillation in the "along-track" (along-orbit) direction that has a period which is one-half the orbital period of the tide-raising body (14.5 days for the moon, 6 months for the sun).<sup>\*</sup> To illustrate this effect and to check once again the numerical consistency of our program, we performed the following experiment.

Using the same set of initial conditions ( $a = 1.17$ ,  $e = 0.0021$ ,  $i = 89.6^\circ$ ,  $\Omega = 339^\circ$ ,  $d = 40$ , 1974), we computed the position of a satellite with and without the moon body-tide. We then differenced the ephemerides at common times and resolved the vector difference in the along-track direction. Results of this simulation, produced for an 8-day ephemeris, are shown in Fig. 12. This curve is the sum of a linear-time function and a 14.5-day sinusoid. The linear term arises because the two orbits do not occupy the same potential surface and consequently have slightly different periods. The sinusoid arises because of the periodic potential field associated with the lunar tide.

Now, to demonstrate the effect of the moon body-tide on navigation we selected a 4-day data span and determined two orbits that fit the data, one with all forces included, one without body-tides. While these orbits agree well with the data (and with each other) over 4 days, they do not agree well when extrapolated to a

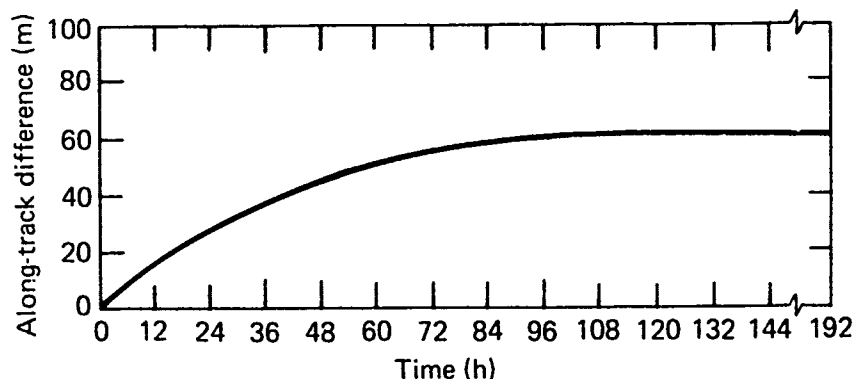


Fig. 12 Expected Along-Track Residuals (lunar body-tide effect).

<sup>\*</sup>If the orbit is redetermined at frequent intervals — frequent meaning closely spaced compared with the lunar period — the tides have almost no effect on the navigation result; no effect even if they are ignored.

total arc of 8 days (Fig. 13). The difference between the two sets of residuals, shown in Fig. 13, should be the same as Fig. 12 after removal of a linear term that best fits the first 4 days of that curve. Verification of this fact can be seen in Fig. 14, a plot of the modified (as described above) simulation results versus the along-track residuals.

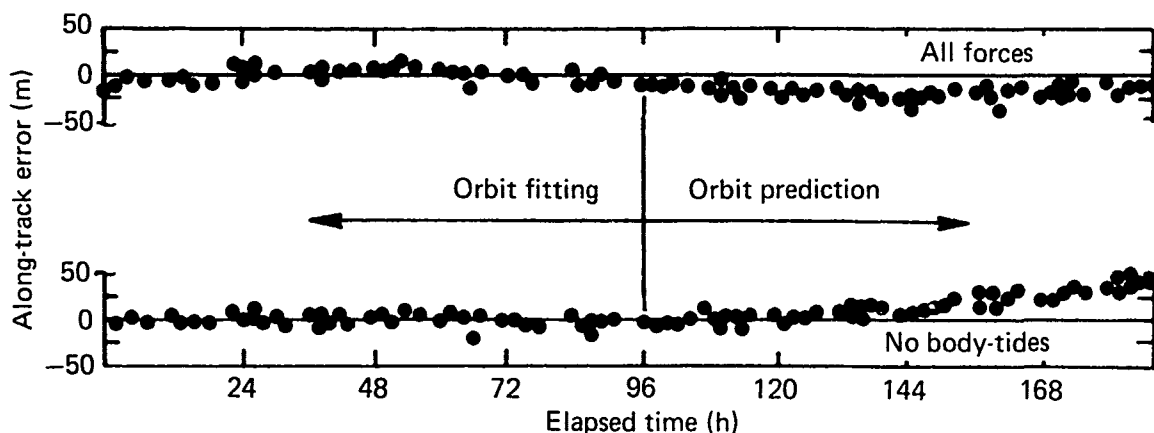


Fig. 13 Orbit Determination With and Without Lunar Body-Tide Effect.

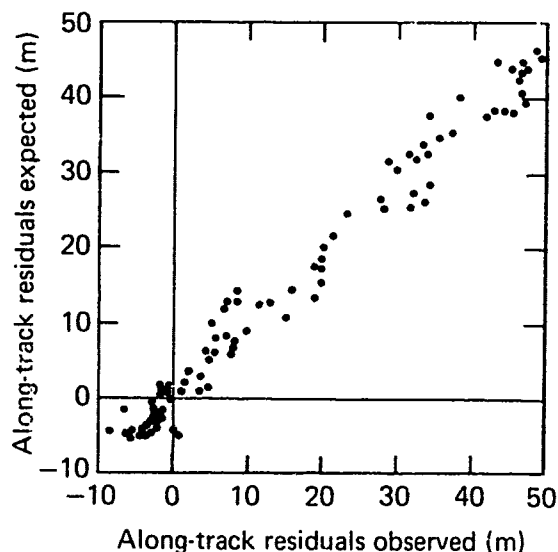
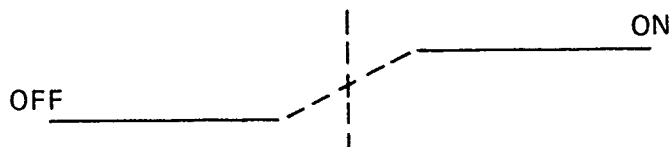


Fig. 14 Comparison Between Observed and Simulated Body-Tide Error.

#### 14. RADIATION PRESSURE

Radiation pressure is a very tricky force to deal with in the numerical integration algorithm used to construct the satellite ephemeris. It is tricky because the force has a discontinuity when the satellite passes into, or out of, the shadow of the earth. This on-off time can occur anywhere with respect to the beginning of the (discrete) integration step. Since forces are evaluated only on the discrete steps, numerical integration errors arise as a consequence. An analysis of the effect (Ref. 38) shows that (on the average) the orbit error grows as the  $3/2$ 's power of (orbit) arc length and at the end of 14 satellite revolutions ( $\approx 1$  day) it is about 2 m (the radiation pressure is about  $1 \text{ dyne/m}^2$ ). To remove this error source, several necessary pieces were assembled.

1. The integration algorithm was shown to give the right answer for a discontinuous force when the discontinuity was replaced with a piece-wise linear function,



providing the length of the ramp was equal to the numerical integration step-size. The obvious was then implemented.

2. With an analytical formulation of the orbit, we keep a running prediction of the shadow-crossing geometry.
3. We center a ramp on the transition point with the length of the ramp equal to the integration step-size.

This sounds quite simple but making the program smart enough to cope with the satellite just grazing the terminator complicates the matter!

---

Ref. 38. W. L. Ebert, "Errors in the Ephemerides of Satellites Caused by Numerical Integration," APL/JHU TG 1233, 1973.

THE JOHNS HOPKINS UNIVERSITY  
APPLIED PHYSICS LABORATORY  
LAWRENCE, MARYLAND

We also designed and programmed a version to include the penumbra/umbra transition and the albedo. These elaborations do not currently affect the derived orbit in any significant way: the orbit frequency term, generated by the albedo, is currently an order of magnitude less than the corresponding term in the geopotential uncertainty.

## 15. DRAG COMPENSATION (DISCOS)

As a result of the Transit improvement program, the new type of satellite to be added to the system has vastly improved hardware (Ref. 39). The satellite is called TIP-II (Fig. 15). One of the new features is DISCOS, which removes the along-track component of drag and radiation forces.

In the DISCOS, a closed loop thrusting system keeps the satellite centered on a "proof mass" that is shielded from the satellite surface forces. As a result, the satellite is constrained to the proof-mass orbit which experiences no drag. An experimental 3-axis system was successfully flown in the TRIAD-I satellite in 1972 (Ref. 21). To simplify the system, the operational version was reduced to a single (along-track) axis compensation. The single-axis system requires precise, 3-axis stabilization. The thrust is supplied by ionized Teflon thrusters.

The presence of DISCOS improves the accuracy (predictability) of the satellite ephemeris. With the drag removed, the ephemeris precision becomes limited by the geopotential model errors. As indicated earlier (Table 6, item 3), this precision will be under 10 m with the WGS-72 geopotential model.

TRIAD-I orbit experiments (Ref. 40) showed that the ephemeris could be extrapolated with the DISCOS for at least a week without appreciable degradation in accuracy. This reduces system operation costs by requiring less frequent orbit computation.

---

Ref. 39. J. Dassoulas, "The TRIAD Spacecraft," APL Technical Digest, Vol. 12, No. 2, pp. 2-13, 1973.

Ref. 40. R. E. Jenkins, "Performance in Orbit of the TRIAD DISCOS System," APL Technical Digest, Vol. 12, No. 2, pp. 27-35, 1973.

THE JOHNS HOPKINS UNIVERSITY  
APPLIED PHYSICS LABORATORY  
LAUREL, MARYLAND

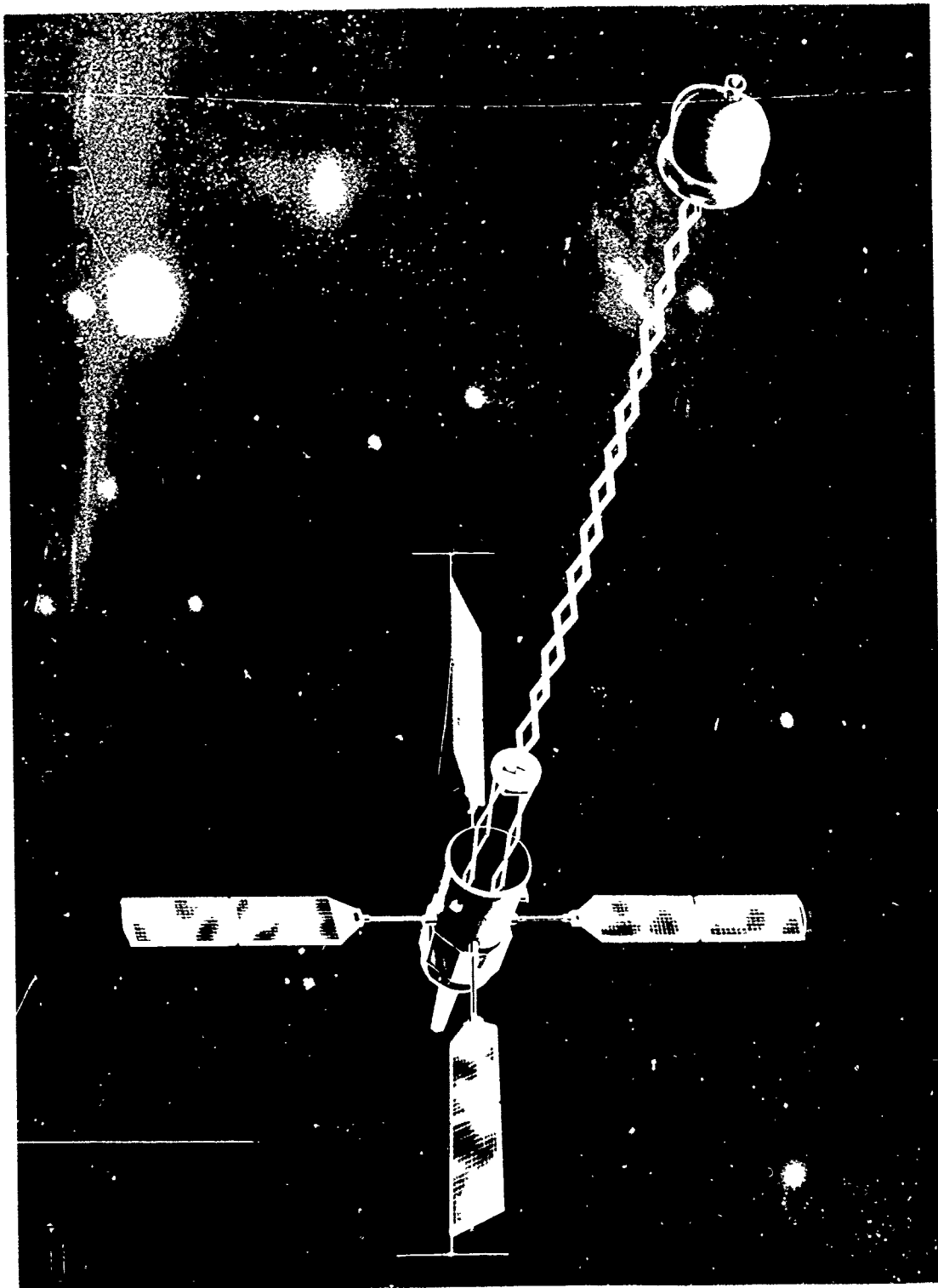


Fig. 15 TIP Satellite, Artist's Concept.



## 16. FLIGHT COMPUTER AND EXTRA MESSAGE

Decreasing the orbit error below 10 m will not immediately benefit the navigator using the broadcast ephemeris. This is true because of the rounding errors (among other reasons), discussed in Section 6. However, there is another hardware feature of the new Transit satellite that provides the necessary flexibility to remove this error source. Included in T1P is a general-purpose computer to replace the hardwired memory and an additional ephemeris modulation that does not affect present receivers.

The flight computer is a 32k, 16-bit-word general processor with a 4.8- $\mu$ s memory access time. The computer is about the ilk of the second-generation computers such as the IBM 7094. The software system currently implemented in the flight computer manages the loading and rebroadcasting of the ephemeris and also controls the satellite clock. Ten days worth of ephemeris can be stored, and the broadcast ephemeris is made to look exactly like the present message. The computer is programmable from the ground.

The important point is that the flight computer permits us to change the message content by reprogramming the management software. (Whatever we do, no mandatory changes will be required of any user.)

The extra ephemeris modulation is a phase modulation that is "transparent to existing receivers." It is superimposed on the normal "double-doublet" ephemeris modulation (Ref. 41). The new modulation has a data rate that is one-half the current 50 bps. The content of the message will be controlled by the flight computer.

---

Ref. 41. R. J. Heins and E. F. Prozeller, "Development of a Compact Ephemeris Recovery System for the AN/SRN-9/PRN Receiver," APL/JHU Quarterly Report C-SQR/74-3, July-September 1974.

## 17. CLOCK IMPROVEMENTS

The new TIP satellite contains hardware that greatly improves the potential precision of the real-time satellite clock. The IPS (incrementally programmable synthesizer) allows precise control over the satellite oscillator output. The IPS system operates as a "programmable black box" and modifies the 5-MHz oscillator frequency according to the transfer function:

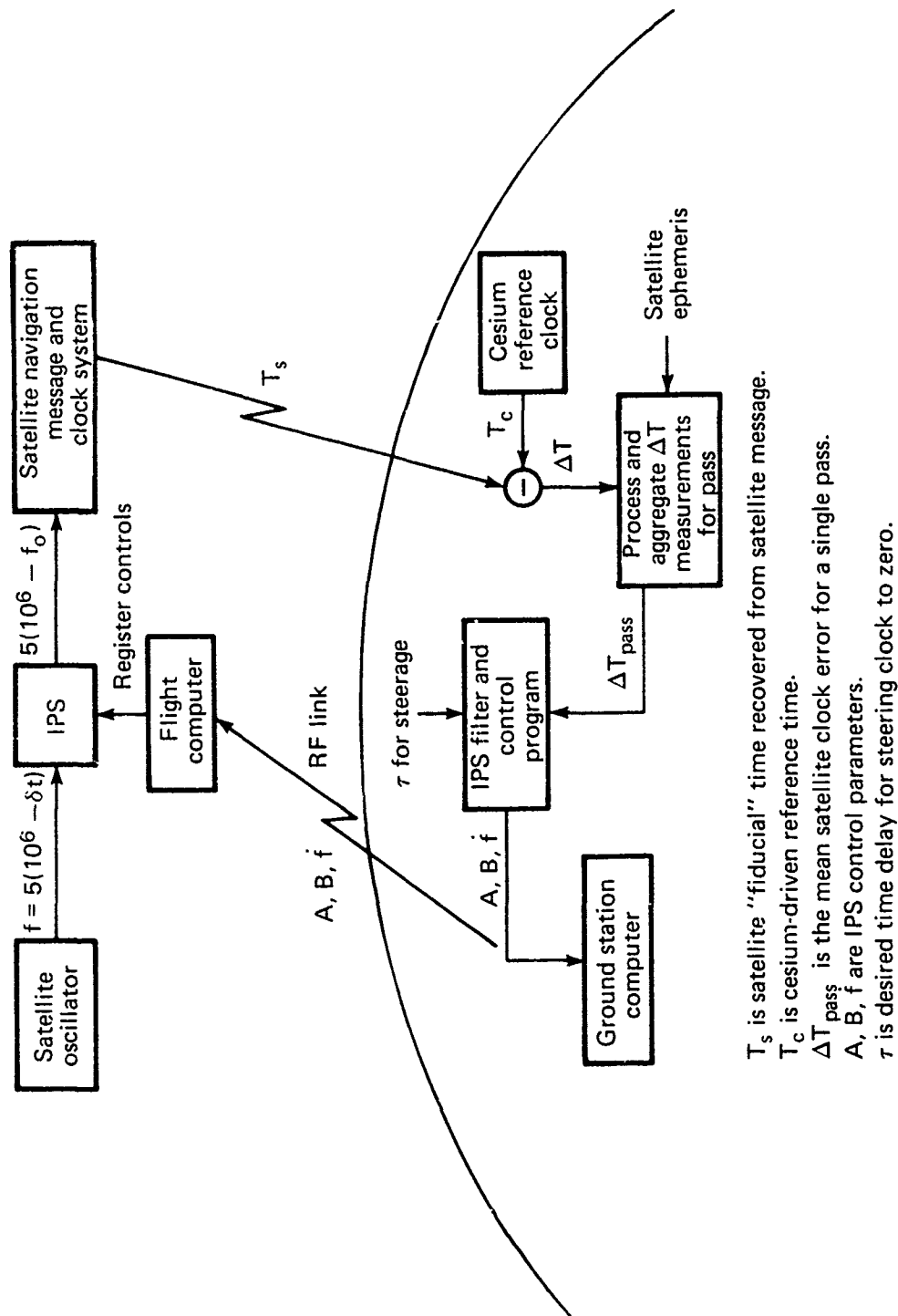
$$f_{\text{out}} = f_{\text{osc}} \left[ 1 - \frac{1}{B} \left( 1 + \frac{1}{A} \right) \right]$$

A and B are control parameters that can be commanded from the ground into the IPS through the flight computer. A and B are on the order of  $10^4$ . A exerts fine control while B is the coarse control. A portion of the real-time software system in the flight computer is devoted to manipulating A and B to correct for oscillator offset, aging, drift, and random jumps.

The system can be used to maintain a high-precision clock by observing the satellite time on the ground and injecting IPS control parameters for clock steering. The closed loop system is shown in Fig. 16. The IPS filter-and-control program is a Kalman filter that recursively processes the clock error measurements and computes the controls (A, B, f) required to steer the clock error to zero over the specified time interval  $\tau$ . The control parameters are then injected into the flight computer from a ground station.

The time constant of the loop is quite long (compared with most control loops) since the measurement and injection rates are set by the times when the satellite is visible. An injection rate of about one a day is the practical lower limit with the current ground system. Generally, the more often the controls can be applied, the more accurately the clock can be held to the ground reference. The one-a-day rate was chosen to accommodate the satellite visibility schedule of the operational ground system.

The oscillator model used in the IPS Kalman filter program is one of a drifting oscillator that undergoes random (white) jumps in drift-rate and random jumps in frequency on the average of once per day. Ground tests of the system using a prototype oscillator worked well with a one-a-day injection cycle when the standard deviation of the random jumps in the filter noise model was set at



$T_s$  is satellite "fiducial" time recovered from satellite message.  
 $T_c$  is cesium-driven reference time.  
 $\Delta T_{pass}$  is the mean satellite clock error for a single pass.  
 $A, B, \dot{f}$  are IPS control parameters.  
 $\tau$  is desired time delay for steering clock to zero.

Fig. 16 IPS Clock Control System.

$10^{-12}$  in frequency and  $2 \times 10^{-12}$  per day in drift. We were able to continuously control the clock error to  $10^{-7}$  s for about 1 week,\* which is testimony to the stability of the oscillator. The ultimate clock accuracy in orbit depends, of course, on the crystal performance, particularly the absence of large random jumps. The IPS and the time-measurement filter program merely allow correction for the observed long-term (longer than a day) random behavior of the oscillator.

All preliminary tests indicate the flight oscillators in the new satellite should perform as well as the prototype. If accurate ground measurements can be made of the clock epoch errors, then the potential exists for 100-ns control.

Another new subsystem provides the capability for satellite clock epoch to be received on the ground with nanosecond precision. This is the pseudorandom noise (PRN) phase modulation that is imposed on the satellite 150- and 400-MHz carriers. This modulation, which is transparent to present navigation receivers, provides a ranging system as well as a time recovery system to any receiver equipped to recover PRN. The two-frequency PRN data can be corrected for first-order ionosphere time delays and the propagation time delay can be determined from the "navigated" slant range.

#### NOTE

The TIP-II satellite was launched on 12 October 1975. The solar (power-generation) panels failed to deploy. A second of the series is scheduled for launch in mid-1976.

---

\*We actually ran the test for over a month, but a week was the longest continuous span we could get without a thunderstorm causing a power loss.

THE JOHNS HOPKINS UNIVERSITY  
APPLIED PHYSICS LABORATORY  
LAUREL MARYLAND

#### ACKNOWLEDGMENT

The writers are indebted to their colleagues in the Space Analysis and Computation Group, in particular, to Messrs. Stanley Dillon and Bruce Holland and Dr. Ward Ebert for their assistance with the computer experiments. Appreciation for supplying statistics on Transit System operation is due the personnel at the Navy Astronautics Group, Point Mugu, CA.

This work was supported by the United States Department of the Navy, Strategic Systems Projects, under Contract N00017-62-C-0604. We acknowledge this support with thanks.

## REFERENCES

1. F. T. McClure, Method of Navigation, U.S. Patent No. 3,172,108, filed 12 May 1958, issued 2 March 1965.
2. W. H. Guier and G. C. Weiffenbach, "Theoretical Analysis of Doppler Radio Signals from Earth Satellites," APL/JHU BB-276, 1958.
3. W. H. Guier and G. C. Weiffenbach, "A Satellite Doppler Navigation System," Proc. IRE, Vol. 48, pp. 407-516, 1960.
4. R. B. Kershner and R. R. Newton, "The TRANSIT System," J. Inst. Nav., Vol. 15, pp. 129-144 (see p. 140), 1962.
5. Navy Astronautics Group, Pt. Mugu, CA, private communication, March 1975.
6. H. D. Black and B. J. Hook, "The Data EDITOR," APL/JHU TG 756, 1966.
7. H. D. Black, "Doppler Tracking of Near-earth Satellites," APL/JHU TG 1031, 1968.
8. L. G. Jacchia, "Static Diffusion Models of the Upper Atmosphere with Empirical Temperature Profiles," Smithsonian Contributions to Astrophysics, Vol. 8, No. 9, pp. 215-257, 1965.
9. B. B. Holland, J. A. Yingling, and M. A. Walko, "The Second Generation Integration Routine (IGC)," APL/JHU TG 466 (Rev.), 1970.
10. A. Eisner, "Atmospheric Density Studies," APL/JHU TG 951, 1967.
11. C. Marvin, "The Operation of the Satellite Clock Control System," APL/JHU TG 523, 1963.
12. W. H. Guier and R. R. Newton, "The Earth's Gravity Field as Deduced from the Doppler Tracking of Five Satellites," J. Geophys. Res., Vol. 70, No. 18, pp. 4613-4626, 1965.
13. T. O. Seppelin, "The Department of Defense World Geodetic System 1972," The Canadian Surveyor, Vol. 28, No. 5, pp. 496-506, Ottawa, Canada, December 1974.

THE JOHNS HOPKINS UNIVERSITY  
APPLIED PHYSICS LABORATORY  
LAUREL, MARYLAND

14. B. B. Holland et al., "Approaching Geodesy in the 1970's," APL/JHU TG 1055, 1969.
15. W. H. Guier and R. R. Newton, "Status of the Geodetic Analysis Program at the Applied Physics Laboratory," APL/JHU TG 652, 1965.
16. R. R. Newton, "The U.S. Navy Doppler Geodetic System and Its Observational Accuracy," Phil. Trans. Roy. Soc. London, Vol. A262, pp. 50-66, 1967.
17. P. C. Sluiter, "Relative Weighting of Satellite Fixes," Proceedings of a Symposium on Marine Geodesy, New Orleans, LA, p. 151 et seq., 3-5 November 1969.
18. G. C. Manning, "The Motion of Ships Among Waves," Chapter 1, Principles of Naval Architecture, II, H. E. Rossell and L. B. Chapman, eds., Society of Naval Architects and Marine Engineers, p. 45, 1942.
19. W. L. Sjogren, "The Ranger III Flight Path and Its Determination from Tracking Data," JPL Technical Report, 32563, 22, Jet Propulsion Laboratory, Pasadena, CA, 15 September 1965.
20. P. B. Esposito and S. K. Wong, "Geocentric Gravitational Constant Determined from Mariner 9 Radio Tracking Data," Jet Propulsion Laboratory, Pasadena, CA, 1972. (Paper presented at the International Symposium on Earth Gravity Models and Related Problems, St. Louis, MO, 16-18 August 1972.)
21. Staff of the Space Department, The Johns Hopkins University Applied Physics Laboratory, and the Staff of the Guidance and Control Laboratory, Stanford University, "A Satellite Freed of All But Gravitational Forces: TRIAD I," J. Spacecraft and Rockets, Vol. 11, No. 9, pp. 637-644, 1974.
22. B. B. Holland, "Uses of Geociever as a Geodetic Instrument," Proceedings of COSPAR XIII, Madrid, Spain, 10-24 May 1972, Akademie-Verlag, Berlin, p. 71, 1973.
23. V. L. Pisacane, B. B. Holland, and H. D. Black, "Recent (1973) Improvements in the Navy Navigation Satellite System," Navigation, Journal of the Institute of Navigation, Vol. 20, pp. 224-229, 1973.
24. G. Gebel and B. Matthews, "Navigation at the Prime Meridian," Navigation, Vol. 8, pp. 141-146, 1971. (Note: A recent re-determination of the connection between the astronomical meridian and the geodetic meridian replaces the 5.64" [given in this paper] with 5.68"  $\pm$  0.168".)

THE JOHNS HOPKINS UNIVERSITY  
APPLIED PHYSICS LABORATORY  
LAUREL, MARYLAND

25. D. E. Smith, F. J. Lerch, and C. A. Wagner, "Gravitations Field Model for the Earth," Proceedings of COSPAR XIII, Madrid, Spain, 10-24 May 1972, Akademie-Verlag, Berlin, 1973.
26. F. J. Lerch, J. E. Brown, J. A. Richardson, and J. S. Reece, "Gravitational Models GEM-5 and GEM-6, 1974," contribution to The National Geodetic Satellite Program Final Report, American Geophysical Union, Washington, DC, to be published August 1975.
27. E. M. Gaposchkin, ed., Smithsonian Standard Earth III, Smithsonian Astrophysical Observatory Special Report 353, 1973.
28. E. M. Gaposchkin, "Earth's Gravity Field to the Eighteenth Degree and Geocentric Coordinates for 104 Stations from Satellite and Terrestrial Data," J. Geophys. Res., Vol. 79, No. 35, pp. 5377-5411, 1974.
29. R. H. Rapp, Numerical Results from the Combination of Gravitimetric and Satellite Data Using the Principles of Least Squares Collocation, Ohio State University Report No. 200, March 1973.
30. R. H. Rapp, Procedures and Results Related to the Direct Determination of Gravity Anomalies from Satellite and Terrestrial Gravity Data, Ohio State University Report No. 211, July 1974.
31. S. M. Yionoulis, F. T. Heuring, and W. H. Guier, "A Geopotential Model (APL 5.0-1967) Determined from Satellite Doppler Data at Seven Inclinations," J. Geophys. Res., Vol. 77, No. 20, pp. 3671-3677, 1972.
32. R. R. Newton, "An Observation of the Satellite Perturbation Produced by the Solar Tide," J. Geophys. Res., Vol. 70, pp. 5983-5989, 1965.
33. R. R. Newton, "Tidal Numbers and Phases as Deduced from Satellite Orbits," APL/JHU TG 905, 1967.
34. R. R. Newton, "A Satellite Determination of Tidal Parameters and Earth Deceleration," Geophys. J. Roy. Astron. Soc., Vol. 14, pp. 505-539, 1968.
35. K. Lambeck, A. Cazenave, and G. Balmino, "Solid Earth and Ocean Tides Estimates from Satellite Orbit Analysis," Rev. Geophys. Space Phys., Vol. 12, No. 3, pp. 421-434, 1974.



36. R. R. Newton, "Applied Ancient Astronomy," APL Technical Digest, Vol. 12, No. 1, pp. 11-20, 1973.
37. B. C. Douglas, S. M. Klosko, J. G. Marsh, and R. C. Williamson, "Tidal Perturbations on the Orbits of GEOS-1 and GEOS-2," NASA Report X553-72-475, 1972.
38. W. L. Ebert, "Errors in the Ephemerides of Satellites Caused by Numerical Integration," APL/JHU TG 1233, 1973.
39. J. Dassoulas, "The TRIAD Spacecraft," APL Technical Digest, Vol. 12, No. 2, pp. 2-13, 1973.
40. R. E. Jenkins, "Performance in Orbit of the TRIAD DISCOS System," APL Technical Digest, Vol. 12, No. 2, pp. 27-35, 1973.
41. R. J. Heins and E. F. Prozeller, "Development of a Compact Ephemeris Recovery System for the AN/SRN-9/PRN Receiver," APL/JHU Quarterly Report C-SQR/74-3, July-September 1974.

BIBLIOGRAPHY

- B-1. R. J. Anderle, "Transformation of Terrestrial Survey Data to Doppler Satellite Datum," J. Geophys. Res., Vol. 79, No. 35, pp. 5319-5331, 1974.
- B-2. D. W. Denzler, J. R. Norton, and J. A. Trennepohl, "TRANSIM — A Low Cost Satellite Navigation System," APL/JHU TG 1204A and TG 1204B, Vols. I and II, 1972.
- B-3. W. H. Guier, "Ionospheric Contributions to the Doppler Shift at VHF from Near-earth Satellites," Proc. IRE, Vol. 49, pp. 1680-1681, 1961.
- B-4. W. H. Guier, "Ionospheric Contributions to the Doppler Shift at VHF from Near-earth Satellites," APL/JHU CM-1040, 1963.
- B-5. W. H. Guier, "Geodetic Problems and Satellite Orbits," Chapt. 8, Lectures in Applied Mathematics, Vol. 6, Space Mathematics, Part II, J. Barkley Rosser, ed., American Mathematical Society, 1966.
- B-6. W. H. Guier, "Satellite Navigation Using Integral Doppler Data: The AN/SRN-9 Equipment," J. Geophys. Res., Vol. 71, No. 24, pp. 5903-5910, 1966b.
- B-7. W. H. Guier, "Data and Orbital Analysis in Support of the U.S. Navy Satellite Doppler System," Phil. Trans. Roy. Soc. London, Vol. A262, pp. 89-98, 1967.
- B-8. W. H. Guier, R. R. Newton, and G. C. Weiffenbach, "Analysis of the Observational Contribution to the Errors of the Navy Satellite Doppler Geodetic System," APL/JHU TG 653, 1965.
- B-9. G. Gutheim, "Program Requirements for Two-minute Doppler Navigation Solution," APL/JHU TG 819-1, 1967.
- B-10. H. S. Hopfield, "Tropospheric Effect on Electromagnetically Measured Range: Prediction from Surface Weather Data," Radio Sci., Vol. 6, pp. 357-367, 1971.
- B-11. H. S. Hopfield, "Two-quartic Tropospheric Refractivity Profile for Correcting Satellite Data," J. Geophys. Res., Vol. 74, No. 18, pp. 4487-4499, 1973.
- B-12. R. E. Jenkins, "A Satellite Observation of the Relativistic Doppler Effect," Astron. J., Vol. 74, p. 960, 1969.

THE JOHNS HOPKINS UNIVERSITY  
APPLIED PHYSICS LABORATORY  
LAUREL MARYLAND

- B-13. R. B. Kershner, "Present State of Navigation by Doppler Measurement from Near-earth Satellite," APL Technical Digest, Vol. 5, No. 2, pp. 2-9, 1965.
- B-14. K. Lambeck, "The Relation of Some Geodetic Datums to a Global Geocentric Reference System," Bull. Geodesique, Vol. 99, No. 1, pp. 37-46, March 1971.
- B-15. R. R. Newton, "The Navy Navigation Satellite System, Space Research VII," Proceedings of the Seventh International Space Science Symposium, Vienna, Austria, pp. 735-763, 1966.
- B-16. J. B. Oakes, "Design of an Ultra-stable Oscillator for Satellites," APL Technical Digest, Vol. 2, No. 1, pp. 10-14, 1962.
- B-17. Staff of the Space Development Department, The Johns Hopkins University Applied Physics Laboratory, Navy Navigation Satellite System User Equipment Handbook, APL/JHU SDO-3100, 1975.
- B-18. A. M. Stone and G. C. Weiffenbach, "Radio Doppler Method of Using Satellites for Geodesy, Navigation and Geophysics," Progress in the Astronautical Sciences, Vol. 1, Chapt. III, pp. 95-144, 1962.
- B-19. S. M. Yionoulis, "The Use of Datellite Doppler Data for Geodetic Purposes," published in Les Methodes Dynamiques de Geodetic par Satellites, Centre National d'Etudes Spatiales, Paris, 1967.
- B-20. S. M. Yionoulis, "Algorithm to Compute Tropospheric Refraction Effects on Range Measurements," J. Geophys. Res., Vol. 75, p. 7636, 1970.
- B-21. S. M. Yionoulis, The Johns Hopkins University Applied Physics Laboratory contribution to The National Geodetic Satellite Program, American Geophysical Union, Washington, DC, to be published August 1975.

## Appendix

### ANNOTATED GUIDE TO THE REFERENCES AND BIBLIOGRAPHY

1. The effects of ionosphere refraction and how this potential error source is removed: B-3, B-4, and B-18.
2. The tropospheric refraction correction: B-10 and B-11; and its computation: B-20.
3. The details of the measurement and how it depends on the navigator's position: B-13.
4. The format and content of the satellite-broadcast ephemeris: B-9 and B-15.
5. The measurement errors and how they are related to the navigation error: Ref. 16 and B-8.
6. The orbit-generation computation: Refs. 3, 6, and 7, and B-7.
7. Error budgets on the overall system: Refs. 16 and 23.
8. The Transit geopotential models and their development: Refs. 12 and 31; B-5, B-19, and B-21.
9. Coordinate systems associated with the fix: Refs. 23 and 24.
10. Currently available military navigation equipment: B-17.
11. Mathematical details of the navigation computation: B-2, B-6, and B-15.
12. Relativistic effects and the formulation of the doppler shift: B-12.
13. The message modulation: B-15.
14. The design of the satellite oscillator: B-16.
15. A low-cost navigation set: B-2.
16. Transformation of fix to local datum coordinates: Ref. 13, B-1, and B-14.

## INITIAL DISTRIBUTION EXTERNAL TO THE APPLIED PHYSICS LABORATORY\*

The work reported in TG 1305 was done under Navy Contract N00017-72-C-4401. This work is related to Task S1, which is supported by NAVSEASYSKOM.

ORGANIZATION	LOCATION	ATTENTION	No of Copies
<b>DEPARTMENT OF DEFENSE</b>			
DDC	Alexandria, VA		12
<u>Department of the Navy</u>			
NAVSEASYSKOM	Washington, DC	SEA-09G3	2
NAVAIRSYSKOM	Washington, DC	AIR-50174	2
NAVPRO	Laurel, MD 20810		1
NSWC, Dahlgren Laboratory	Dahlgren, VA 22448	R. J. Anderle	1
Strategic Systems Project Office	Washington, DC 20376	SP-24	1
<b>FOREIGN GOVERNMENT AGENCIES</b>			
Space Applications Centre Geodesy Section (RSA) Government of India	Post Bag No. 11 Jodpur Tekra Ahmedabad 380009	Dr. M. R. Sivaraman	1
Space Applications Centre Geodesy Section (RSMD) Government of India	Sahajanand College Building Ahmedabad 380015 India	A. N. Ramanathan, Head	1
Ministry of Defence Royal Aircraft Establishment	Farnborough Hants, England	Dr. Desmond King-Hele, F.R.S. Procurement Executive	1
Geodetic Survey of Canada Surveys and Mapping Branch Department of Energy, Mines and Resources	615 Booth St. Ottawa, Canada	Dr. J. Kouba	1
<b>UNIVERSITIES</b>			
The University of Nottingham Department of Civil Engineering University Park	Nottingham NG7 2RD England	Dr. V. Ashkenazi	1
Department of Surveying Engineering University of New Brunswick	Fredericton, New Brunswick Canada	Mahamed M. Nassar	1
The Ohio State University Research Foundation	Columbus, OH 43212	Department of Geophysical and Geodetic Science	1
<b>COMPANIES</b>			
Continental Shelf Division Royal Norwegian Council for Scientific and Industrial Research (NTNF)	Hoffsveien 13 Oslo, Norway	Mr. Thv. Mellingen	1
Geodetic Institute Geodetic Department 1	Gamlehve Alle 22 DK-2920 Charlottenlund Copenhagen, Denmark	Dr. Frede Madsen	1
Maritime Institute	Gdansk 80-830 Poland	Professor T. Jednorzal, Director	1
Polytechnic Engineering Ltd.	Long March Industrial Estate Daventry, England	Dr. Roger Jarvis Mr. Michael J. Perry	1 1
JMR Instruments, Inc.	20621 Plummer St. Chatsworth, CA 91311	Paul D. Rodgers Director of Marketing	1
Gulf Research and Development Co.	P.O. Drawer 2038 Pittsburgh, PA 15230	Mr. H. U. Thrunheer	1
The Magnavox Company	2829 Maricope St. Torrence, CA	Thomas A. Stansell, Jr.	1
Satellite Positioning Corp.	6614 Hornwood Drive Houston, TX 77036	James G. Morgan	1
	10 Diomidous St. Argos, Greece	CDR H. C. G. Georgios Thanopoulos	1
Requests for copies of this report from DoD activities and contractors should be directed to DDC, Cameron Station, Alexandria, Virginia 22314 using DDC Form 1 and, if necessary, DDC Form 55.			

\*Initial distribution of this document within the Applied Physics Laboratory has been made in accordance with a list on file in the APL Technical Publications Group.